Action of differential rotation on the large-scale magnetic field of stars and planets

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Abstract.
Magnetic fields are present in many different astrophysical objects, such as accretion discs, stars, and planets. They influence the evolution and dominate the interior dynamics of these objects, in particular their evolutionary stages. The presence of a weak (subthermal) magnetic field plays a crucial role to drive turbulence in accretion discs thus leading to the stresses needed for accretion and angular momentum transport. This instability is known as the MagnetoRotational Instability (MRI) and it has been studied intensively for the last two decades. Recent numerical results show the importance of understanding the dynamo process in accretion discs. Small-scale dynamo action could prevent the saturation of MRI modes whereas the generation of large-scale magnetic fields provides a suitable coherent field for the angular-momentum transport by MRI modes. Observations show a huge variety of stellar and planetary magnetic fields. Cosmic magnetic fields differ in their magnitude, topology and time dependence. Of particular interest is the understanding of cyclic field variations, as known from the Sun. They are often explained by an important Ω effect, i.e., by the stretching of field lines because of strong differential rotation. We computed the dynamo coefficients for an oscillatory dynamo model with the help of the so-called test-field method. We argue that this example is of $\alpha^2\Omega$-type and here the Ω-effect alone is not responsible for its cyclic time variation. More general conditions which lead to dynamo waves in global direct numerical simulations are presented. Zonal flows driven by convection in planetary interiors may lead to secondary instabilities. We showed that a simple, modified version of the MRI (so-called MS-MRI) can develop in the Earth's outer liquid core (Petitdemange, Dormy, Balbus, GRL,35, 2008). The force balance in the Earth's core and in classical astrophysical applications of the MRI (such as gaseous discs around stars) is different. The weak differential rotation in planetary interiors yields an instability by its constructive interaction with the much larger rotation rate of the planets. The resulting destabilizing mechanism is just strong enough to counteract the stabilizing resistive effects, and to produce growth on geophysically interesting time scales. MRI and dynamo action are both interesting in their own right, however, their interaction is crucial in order to understand the dynamics of accretion discs, stellar and planetary interiors.

Keywords: MHD instabilities, dynamo action, MHD waves
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INTRODUCTION

Magnetic fields are present in a wide variety of astrophysical objects: planets, stars, accretion disks, Active Galactic Nuclei .... For instance, they are known to influence all evolutionary stages of stars from collapsing molecular clouds which induce stars and planets, to supernovae, degenerate white dwarfs and neutron stars. However, if their

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impotance is of primary interest, the dynamics and even the origin of magnetic fields are still opened questions. Dynamo action must take place in order to maintain the magnetic activity against the ohmic dissipation for objects having moderate magnetic Reynolds numbers as planetary interiors. Even if the dissipation coefficients are weak in stellar interiors, we have to understand the physical mechanisms responsible for the observed cyclic time variations of the solar magnetic field. Thanks to the recent progress in detection, a wide variety of magnetic activity is observed (Donati & Landstreet 2009).

The ordered magnetic activities in the universe (for galaxies clusters, galaxies, stars, planets…) originate from dynamo action. In the mean-field formalism, differential rotation can be an important ingredient of this process through the Ω effect. The Ω effect consists of a distortion of poloidal magnetic field lines into toroidal components. The dynamo loop is closed by the α effect which generates polodial fields from the toroidal fields if the velocity field is complex enough (Moffatt 1978). This particular dynamo loop is termed $\alpha\Omega$ dynamo and it is thought to occur in differentially rotating systems. More subtle mechanisms induced by a large scale differential rotation exist (Brandenburg & Subramanian 2005, Rädler & Bräuer 1987).

In astrophysical objects, the Reynolds numbers associated with the flows are so huge that turbulence develops and induces an important random flow having no coherence. However, many examples show that an additional coherent flow is induced by particular constraints. Thin accretion disks are differentially rotating systems where the massive central object imposes its gravitational potential. As a result, the rotation law in the accretion disk is close to the Keplerian rotation law (see King & Raine's book). The solar differential rotation is known in great detail from helioseismology and serves as an important observational constraint for simulations of thermal convection in rotating spherical shells. The solar differential rotation profile can be directly deduced from a force balance between Coriolis force, pressure gradient and buoyancy term (thermal wind balance). Balbus *et al.* (2009) provide theoretical arguments in order to explain the observed rotation profile in the solar convective zone (see also Küker *et al.* 2011). Coherent flows also develop in planetary interiors which are rapidly rotating systems strongly affected by the Coriolis force. Convective motions develop in the form of elongated columns (see e.g. Busse 2002). In the geophysical context, Petitdemange *et al.* (2008) (noted PDB08) showed that even if the differential rotation is weak, it can drive seondary MHD instability in the Magnetostrophic regime which is relevant in planetary interiors.

We focus here on the action of radial differential rotation. Either the angular velocity decreases outwardly ($d\Omega/ds < 0$, as it does in Keplerian accretion disks) or it increases with the radius s ($d\Omega/ds > 0$) as it does in the equatorial plane of the solar convective zone. In both cases, the differential rotation induces an Ω effect. But if $d\Omega/ds < 0$, a weak magnetic field can have a crucial role and trigger turbulence. In this manuscript, we highlight successively the action of the differential rotation in planetary and stellar interiors when $d\Omega/ds < 0$ and when $d\Omega/ds > 0$.

Many years of numerical simulations of accretion disks have allowed us to improve our understanding of turbulence driven by the Magnetorotational Instability (MRI). This MHD instability was developed by S. Balbus & J. Hawley (1991) for its crucial role in Keplerian accretion disks. In these systems, shear flow results from a rotation law imposed by the massive central object. The non-linear development of this instability has been extensively studied (see Balbus & Hawley 1998, Balbus 2003 for useful reviews). Dynamo activity in turbulence driven by the MRI has been studied by Hawley, Gammie, Balbus 1996 and is now an active subject of research. In general, the MRI is widely viewed as the most likely origin for the turbulence that is needed to allow accretion to proceed in astrophysical disks. But disks are not the only MRI venue. The MRI results from the destabilizing action of a weak magnetic field on an outwardly decreasing angular velocity field, and these conditions are very common. They ensure that the MRI is relevant not only for our understanding of accretion disks, but for many other objects of widely different length scales as well. On the largest scales, the MRI has been invoked to explain turbulence in HI gas in galaxies (Sellwood & Balbus 1999). On the smallest scales, the MRI may be involved with influencing the magnetic activity in planetary cores (Petitdemange *et al.* 2008). In stellar interiors, Arlt *el al.* (2004) argued that the MRI could act to reduce the large-scale shear.

SHEAR AND CONVECTION IN SPHERICAL SHELL

Approximations and equations

In this manuscript, we present numerical results obtained in solving MagnetoHydrodynamic (MHD) equations in the Boussinesq approximation

$$\begin{aligned} E\left(\partial_t \mathbf{u} + (\mathbf{u}\cdot\nabla)\mathbf{u}\right) &= -\nabla\pi + E\Delta\mathbf{u} - 2\mathbf{e_z}\times\mathbf{u} && (1)\\ & \quad + Ra\frac{\mathbf{r}}{r_0}\theta + P_m^{-1}(\nabla\times\mathbf{B})\times\mathbf{B} && (2)\\ \partial_t\mathbf{B} &= \nabla\times(\mathbf{u}\times\mathbf{B}) + P_m^{-1}\Delta\mathbf{B} && (3)\\ \partial_t\theta &= -\mathbf{u}\cdot\nabla(\theta+T_s) + P_r^{-1}\Delta\theta && (4)\\ & \nabla\cdot\mathbf{u} = \nabla\cdot\mathbf{B} = 0 && (5)\end{aligned}$$

where the Ekman number, the Rayleigh number, the magnetic Prandtl number and the thermal Prandtl number are used and define respectively as

$$E = \frac{\nu}{\Omega_0 D^2}, \qquad Ra = \frac{\alpha_T g \Delta T D}{\nu\Omega_0} \qquad \text{and} \qquad P_m = \frac{\nu}{\eta}. \qquad P_r = \frac{\nu}{\kappa} \tag{6}$$

The unit vector $\mathbf{e_z}$ indicates the direction of the rotation axis and gravity varies linearly with the radius $\mathbf{r}$. ν denotes the kinematic viscosity, κ the thermal diffusivity, α_T the thermal expansivity and g gravity at the outer radius R_o, Ω_0 the rotation rate, D the shell width, ΔT stands for the temperature difference between the spherical boundaries. The classical model for geodynamo simulations (Christensen *et al.* 2001) consists of convection driven by an imposed temperature gradient between the inner and the outer shell boundary, at which the temperature is fixed. The mechanical boundary conditions are no-slip at the boundaries. Moreover, the magnetic field is assumed to continue as a potential field outside the fluid shell. In our model, we use these equations for the velocity $\mathbf{u}$, the magnetic field $\mathbf{B}$ and the temperature perturbation θ with the help of the PaRoDy code (Dormy *et al.* 1998).

The Boussinesq approximation consists of ignoring the density variation except in the buoyancy term. This approximation seems to be justified in planetary interiors as in the iron liquid of the Earth's outer core where the density stratification covers less than one scale height and the fluid can be treated as incompressible. The Boussinesq approximation seems to fail in stellar interiors where the anelastic approximation takes the density stratification of the gas into account. However, we do not consider such compressible effects in this manuscript. The main motivation for considering the Boussinesq approximation is that stellar and planetary interiors must share some fundamental physical properties leading to the maintainance of the magnetic activity. In addition, 15 years of active research on Boussinesq simulations has shown their ability to reproduce numerically many features of the Earth's magnetic field and planetary fields (Christensen *et al.* 1998, 1999; Busse *et al.* 1998, Takahashi *et al.* 2005, Christensen & Wicht 2007, Christensen 2009, Christensen 2011). Possible field generation mechanisms in planetary conducting zones have been identified and investigated by Olson *et al.* (1999). Progress both in numerical methods as well as in parallel machine architecture has made it possible to explore an extensive parameter space and to determine scaling laws for some features of planetary fields (Christensen & Aubert 2006, Christensen *al.* 2009). Among them is the magnetic field strength, which is predicted to scale with the available energy flux to the power of 1/3, independent of rotation rate. Surprisingly, this scaling law also explains the field strenght observed for rapidly rotating low-mass stars. This finding supports the validity of considering Boussinesq simulations for studying stellar dynamos.

Considering the Boussinesq approximation allows us to compare our results with the state of research of geodynamo numerical models. In addition, ignoring compressible effects except in the buoyancy term reduces the complexity of MHD equations, which is desirable from a numerical point of view.

Zonal flows in rapidly rotating spherical shell

The Earth's core and planetary interiors are, by comparison to accretion disks, relatively small objects, in which resistive effects are on an equal footing with dynamical processes. The rotation properties of the Earth also significantly differ from those of an accretion disk. To leading order, they correspond to solid body rotation, with only a weak differential rotation. Planetary interiors are rapidly rotating systems where the Coriolis force plays an important role. The dimensionless parameters provide a good estimate of dynamical proceses. For the Earth's outer core, the Ekman number is so small ($\approx 10^{-15}$) that, at leading order, the viscous term and the inertial term do not enter in the force balance. The Coriolis force is only balanced by the pressure gradient term, the Lorentz force and the buoyant term.

Theoretical constraints result from this particular force balance taking place in planetary interiors. Taylor's constraint (Taylor 1964) can be interpreted as that the net magnetic torque on each cylinder must vanish. The Coriolis force acts to organize the flow and make it constant on cylinders parallel to the rotation axis (geostrophic or zonal flow). This constraint is known as the Proudman-Taylor constraint (Proudman 1956, Stewartson 1966). The Lorentz force and the Buoyancy force can break down this constraint. But, analytical (Busse 1975) and numerical (Olson *et al.* 1999, Grote & Busse 2001, Christensen 2002, Aubert 2005) studies in agreement with observations (Paris & Hulot 2000, Finlay & Jackson 2003) showed that the geostrophic constraint is robust in the Earth's outer core. Besides surface flow reconstructions, the only (indirect) observational evidence for such shear is inferred from seismological data that have been interpreted as rotation of the solid inner core at a rate of about 0.15 degrees per year relative to the mantle (Vidale *et al.* 2000). There are no observational constraints on the way the corresponding jump in angular velocity is actually accomodated by the flow (spread across the whole core radius or localised in a narrow shear layer).

The excitation of differential rotation in simulations of thermal convection in rapidly rotating, spherical shells has been studied in detail in order to explain the patterns of prograde and retrograde zonal flows observed at the surface of giant planets (Christensen 2002, Jones & Kuzanyan 2009). In these studies, stress free mechanical boundary conditions have been applied. Differential rotation occurs in the form of differentially rotating cylinders concentric to the rotation axis. An eastward zonal flow near the equator accompanied by westward jets at higher latitudes is a robust feature found in these simulations. Christensen (2002) identified a dynamical regime where viscosity is negligible and the force balance is mainly among the Coriolis, inertia and buoyancy forces. Furthermore, he derived scaling laws in this regime, predicting the strength of the mean zonal flow depending on the available heat flux. Although compressibility is important in the hydrogen envelops of giant planets, these scaling laws derived in the Boussinesq approximation predict the observed order of magnitude of zonal flows at their surfaces rather well.

THE MAGNETOSTROPHIC-MRI

In this section, we focus on the action of an outwardly decreasing angular velocity profile on the planetary fields.

MRI in planetary interiors: linear development

In Petitdemange, Dormy & Balbus (2008) (hereafter noted PDB08), it is shown that a simple modified version of the MRI can develop in a fluid environment modeled on the geodynamo. They carried out a local linear calculation of the problem in the resistive, Magnetostrophic regime, in which the Coriolis force is predominantly balanced by the magnetic Lorentz force over the course of the development of the instability, and resistive losses are important in the induction equation. As noted by Acheson & Hide (1973), the magnetostrophic regime is very particular: taken separately, rotation effect and magnetic tension each has a stabilizing effect, but together these proccesses destabilize one other. The result is the MS-MRI.

According to Roberts & Gubbins (1986), the toroidal component could exceed the poloidal one by a factor R_m in the Earth outer core ($R_m \approx 100$), where R_m is the magnetic Reynolds number. The fact that the toroidal component may largely exceed the poloidal

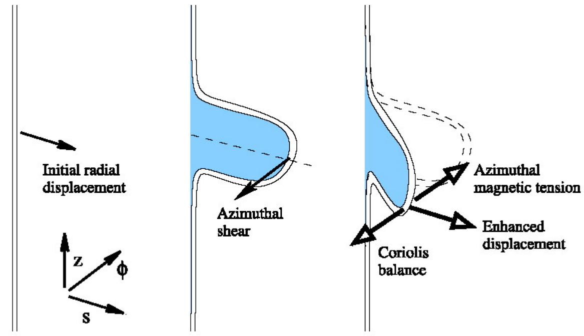

FIGURE 1. Development of the magnetostrophic MRI. (a) Unperturbed field line. (b) Field line is distorted by a radially outward displacement, and subject to velocity shear. (c) Field line develops azimuthal tension which is immediately compensated by the Coriolis force. This compensating force requires a further displacement in the same sense of the initial outward radial extension, and the instability proceeds.

one in planetary interiors, led previous linear stability analyses to be mainly performed in the simplified case of a purely toroidal applied field (Acheson & Hide 1973, Fearn 1983a,b, Fearn 1984, Fearn 1985, Fearn 1988, Fearn 1994). But in previous calculations the emphasis has been upon purely azimuthal fields, non-axisymmetric disturbances, and magnetic instabilities. By contrast, in PDB08, the magnetic coupling is to the poloidal field components, axisymmetric disturbances are front and center, and the instability, while relying on the presence of a magnetic field, has its seat of free energy entirely in differential rotation.

In planets, the classical MRI manifests itself in a new parameter regime, which we have dubbed the magnetostrophic MRI (MS-MRI). This results of an initial local WKB study has been confirmed by linear numerical simulations. The physical mechanism of the MS-MRI is visible on the figure 1 In contrast with accretion disks, the rotation profile in planetary interiors is very close to solid body rotation, the shear is very small. This coupled with high resistivity, may suggest that planetary interiors are not promising candidates for the MRI. But even with a relatively small amount of differential rotation, it was found that the instability can occur. The figure 2 shows the linear development of the MS-MRI with the characteristic phase differences between the radial velocity and magnetic field components clearly visible. It is thought that the MS-MRI could, in principle, explain some rapid variations of the magnetic field such as magnetic impulses (or “jerks”), as well as the classical magnetic secular variation.

The MS-MRI needs the emergence of shear to exist. Convection instability is the primary instability which develops zonal flows and a helical velocity field which gives rise to dynamo action. In this context, the MS-MRI appears as a secondary instability. Analytical and numerical studies of MS-MRI modes have not taken into account the possibility of dynamical coupling between MS-MRI modes and the heat transport. Only the action of convection has been considered accross the presence of differential rotation. The Buoyancy force and the temperature equation have been ignored.

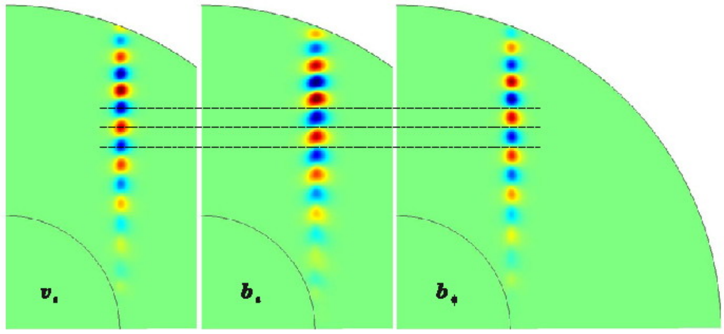

FIGURE 2. Snapshot of the most unstable mode in a direct simulation with $E = 510^{-7}$, $\Lambda = 3$ and $P_m = 0.5$. The physical mechanism of the magnetostrophic-MRI can be traced in the phase shifts between v_s, b_s and b_ϕ: b_s is a quarter-period ahead of v_s, while b_ϕ is exactly out of phase by half a period with b_s.

Action of axisymmetric MS-MRI modes in planetary interiors

A nonlinear follow-up axisymmetric calculation is presented in Petitdemange (2010). After linear perturbations increase exponentially with time, saturation occurs by a modification of the background shear rate. In the context of the classical MRI, this result is somewhat novel: the rotation profile of an accretion disks is always very close to Keplerian. But in the geophysical context, it is well-known that MHD effects can create a geostrophic flow (Braginsky 1975, Fearn & Proctor 1983; Fearn 1994, Jault 1995). We observe numerically the development of such a geostrophic flow when the initial small perturbations reach a certain critical magnitude. This flow progressively reduces the initial sheared velocity field and produces saturation of the MS-MRI. The shear rate then continues to decrease until the system becomes marginally stable (see the figure 3). In fact, different MS-MRI modes develop successively, each saturating by incrementally decreasing the shear. In this sense, the MS-MRI may even be able to regulate differential rotation in planetary cores.

Petitdemange (2010) illustrated this mechanism, motivated by the banded structure observed in Jupiter's atmosphere (Porco *et al* 2003, Cho and Polvani 1996). In particular, he argued against the possibility of an origin of these structures in deep layers, where the fluid is electrically conducting (metallic hydrogen), and therefore influenced by the presence of a magnetic field (Yano 1994, Morin & Dormy 2005, Christensen et al 2001, Aurnou & Olson 2001). Numerical simulations of convection (Heimpel *et al.* 2005) showed that the zonal wind resembles the banded structure of Jupiter and Saturn for high enough Rayleigh numbers. However, such a banded structure in the inner conducting region would be unstable in the presence of vertical magnetic field. Indeed, considering the θ-profile of angular velocity observed in the Jovian atmosphere, and assuming a geostrophic sturcture ($\Omega_{Jup}(s) = U_0(s)/s$ where s is the cylindrical radius), linear analysis predicts that all bands are locally unstable when a vertical magnetic field is imposed at sufficiently small Ekman numbers. Realistic Ekman number can however not be achieved. Instead, Petitdemange (2010) solved the linearized MHD equations with $\Lambda = 1$ and different Ekman numbers. For small enough Ekman numbers (smaller than $5 \cdot 10^{-7}$, he showed that the MS-MRI can destabilize all bands.

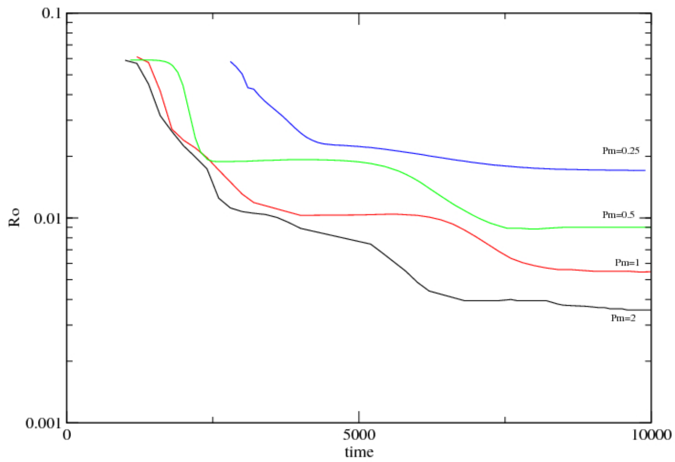

FIGURE 3. Temporal evolution of the maximum R_o for different P_m. The other parameters are $E = 10^{-5}$, $\Lambda = 1$ and $A = 0.01$.

In the nonlinear regime, the bands are progressively suppressed. The forced velocity field used for this simulation has the form $U_0(s) = s\Omega_{Jup}(s)$. The boundary conditions and the initial conditions are unchanged. The simulation has been terminated when saturation starts to set in. The profile has been modified within a very short time period. In this context, the MS-MRI could provide a new constraint on the flow. This would suggest that the observed Jovian banded structure cannot be maintained in the conducting zone, because it would then be unstable to the MS-MRI. This result suggests that either the origin of the banded structure is not deep, or convection is powerful enough to maintain the profile against this destabilizing process. In the latter case, the system remains unstable, and the MS-MRI could then play an important role in the dynamo process. As mentioned in PDB08, this could induce rapid variations of the magnetic field.

ACTION OF LARGE-SCALE ZONAL FLOWS IN OSCILLATORY DYNAMO MODELS

In this section, we investigate the role of a large-scale zonal flow for dynamo action. We consider the full Boussinesq MHD equations and we are in particular interested in understanding the mechanism of oscillatory dynamos. Their origin as well as the reason for their time variation are still open questions of primary importance in order to understand recent stellar magnetic observations (Donati & Landstreet 2009, Morin *et al.* 2010).

Via the Lorentz force, the magnetic field acts back on the flow and the dynamo problem is intrinsically nonlinear. This is the reason why we focus our attention on saturated dynamos obtained in direct numerical simulations.

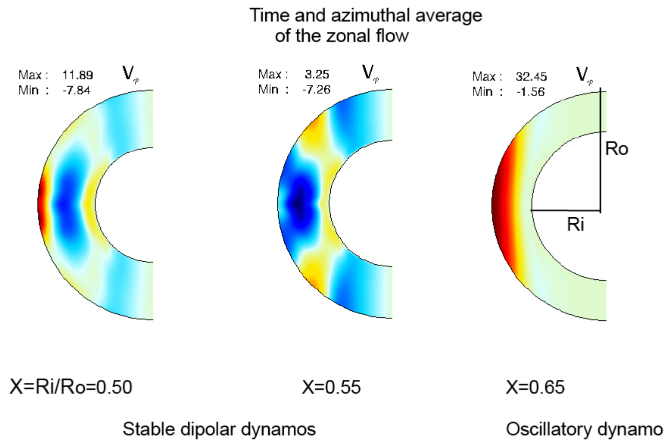

FIGURE 4. Azimuthally and time-averaged zonal flows for models with different aspect ratios (Ri/Ro).

A transition from steady (Earth-like) to oscillatory (Sun-like) dynamos in direct numerical simulations can be obtained by decreasing the width of the convection zone: Goudard & Dormy (2008) found oscillatory models with high aspect ratio. Schrinner, Petitdemange, Dormy (2011), followed their approach and analysed the dynamo mechanism for these oscillatory models. In particular, the question whether an Ω effect is responsible for the cyclic variation of the magnetic field was addressed. The magnetic field in these simulations turned out to be weak, small scaled and of either dipolar or quadrupolar symmetry. Traditionally, the dynamo mechanism is thought to be different for oscillatory and steady dynamos. For both types of dynamos, the generation of poloidal field results from the interaction of helical turbulence with the toroidal field (α effect). Most steady, dipolar dynamos are classified as α^2 dynamos, i.e. the generation of toroidal field from poloidal field results from the same α effect (Olson *et al.* 1999, Schrinner *et al.* 2007). But, for oscillatory dynamos, the generation of toroidal field is believed to originate from the Ω effect.

However, a strong differential rotation is not a necessary condition for oscillatory dynamos as has been demonstrated in several papers presenting models based on the mean-field approach (see e.g. Rädler & Bräuer 1987; Baryshnikova & Shukurov 1987; Schubert & Zhang 2000; Rüdiger *et al.* 2003; Stefani & Gerbeth 2003; Mitra *al.* 2010). The success of mean-field models in reproducing solar-like variations of the magnetic field relies partly on the large number of free parameters, i.e. on the arbitrary determination of the dynamo coefficients. In Schrinner *et al.* (2011), we determined the dynamo coefficients for the first time from direct numerical simulations with the help of the test-field method (Schrinner *et al.* 2005, 2007, Schrinner 2011). The application of the obtained dynamo coefficients in a mean-field model reveals their importance for the generation of the magnetic field.

Our model is set up as described in Goudard & Dormy (2008). Only the mechanical boundary condition at the outer sphere is modified compared to classical geodynamo simulations in order to mimic the stress-free condition at the solar surface. The dimensionless parameters were fixed at: $E = 10^{-3}$, $P_m = 5$, $R_a = 100$ and $P_r = 1$, and different aspect ratios X were considered (the aspect ratio X is defined as the ratio of the inner Ri to the outer Ro shell radius). We reproduced the results obtained by Goudard & Dormy (2008) and observed a transition from a steady, dipolar (Earth-like) dynamo to an oscillatory (Solar-like) dynamo by increasing the aspect ratio. The corresponding mean flows are shown in figure 4. A large scale geostrophic differential rotation is obtained for the oscillatory dynamo with an aspect ratio of $X = 0.65$ wheras the mean flows corresponding to steady and dipolar dynamos (for $0.35 \leq X < 0.65$) do not exhibit contours of the mean zonal flow constant on cylinders. This result could suggest that the differential rotation is directly responsible for the oscillatory behavior. The determination of the equivalent dynamo coefficients allows us to reveal the physical mechanisms responsible for the dynamo loop.

A mean-field calculation based on the dynamo coefficients and the mean flow determined from the self-consistent model is presented in the bottom line of figure 5. The fastest growing eigenmodes form a conjugate complex pair and give rise to a dynamo wave that nicely compares with the direct numerical simulations. We found that the oscillatory dynamo is of $\alpha^2\Omega$ type. It means that the Ω effect induced by the differential rotation is not directly responible for the observed oscillatory behavior. If we artificially suppress the Ω effect in a mean field calculation, the oscillatory behavior persists. Surprisingly, the corresponding $\alpha\Omega$ dynamo is, however, steady and dipolar.

Moreover, it is shown in the figure 5 that the calculation based on the mean field coefficients produces a butterfly diagram which is consistent with the corresponding direct numerical simulation. This indicates that the physical mechanisms responsible for the oscillatory behavior in the direct numerical simulation are well captured by the mean field calculation.

The frequency and the propagation direction of the dynamo wave visible in figure 5 strongly depend on the differential rotation, in agreement with Busse & Simitev (2006). Schrinner *al* (2011) follow their approach and give an estimate for the cycle frequency by applying Parker's plane layer formalism (Parker 1955). We highlighted that the magnitude of the differential rotation determines the frequency and the propagation direction of the resulting dynamo waves.

CONCLUDING REMARKS

The study of the action of large-scale radial shear on magnetic activity of stars and planets is of primary interest in order to understand the observations. When the angular velocity increases outward, MRI modes can develop even if the shear rate is weak in planetary interiors. Such modes are not responsible for dynamo action but they could induce short temporal evolution of the magnetic field at the Earth's surface or influence the secular variation.

Nonlinear simulations revealed that MRI modes in planetary interiors could regulate the angular momentum distribution in the dynamo zone. Convection creates large-scale zonal flows in rapidly rotating systems as planets and low-mass stars. If the radial shear becomes strong enough, MRI modes develop and act to decrease the large-scale shear rate. The action of MRI in planetary interiors has only been investigated so far using

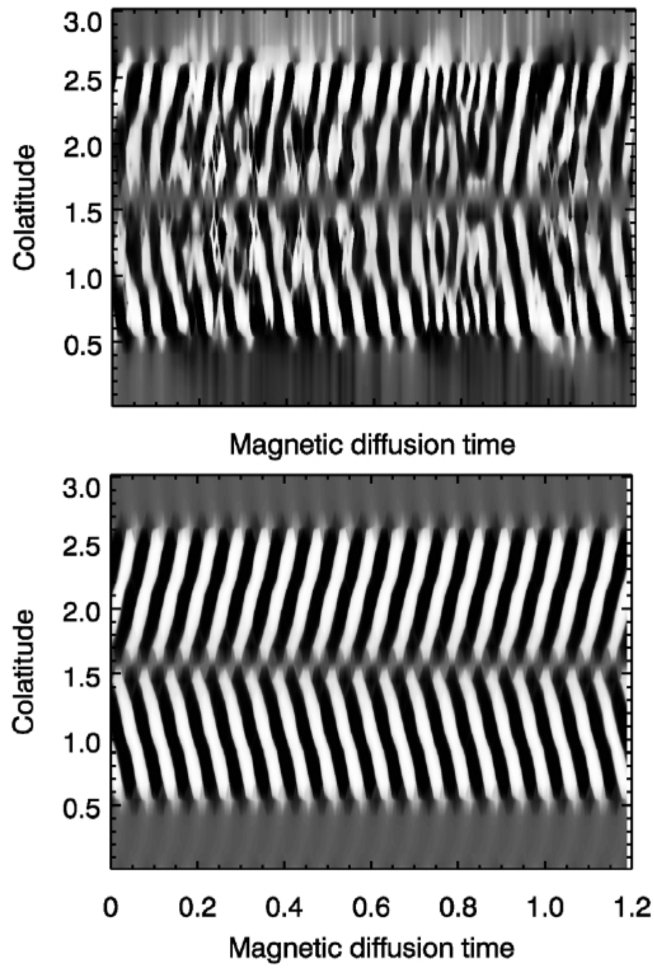

FIGURE 5. Azimuthally averaged radial magnetic field at the outer shell boundary varying with time (butterfly diagram) resulting from a selfconsistent calculation (top) and mean-field calculation (bottom). The contour plots were normalised by their maximum absolute value at each time step considered. The colour-coding ranges from -1, white, to $+1$, black.

very simple analytical and numerical models (Petitdemange *et al.* 2008, Petitdemange 2010) and additional studies using more realistic models, considering axisymmetric and non-axisymmetric MS-MRI modes, are desirable and in preparation.

MRI modes could also develop in stellar interiors under some particular configurations. However, the angular velocity profile in the solar convection zone, as shown by heliosismology is not suitable for such an instability as a global process. In the equatorial plane, the angular velocity increases with radius. In this context, it is essential to investigate the action of large-scale shear on dynamo action.

A particular dynamo mechanism does not seem to be responsible for the occurrence of periodically time-dependent magnetic fields. It turns out that the influence of the large-scale radial shear (the Ω effect) is not necessary for cyclic field variations. Instead, the action of small-scale convection turns out to be essential. For the model presented in Goudard & Dormy (2008) and in Schrinner *el al.* (2011), small convective length scales are forced by a thin convection zone. Additional investigations are needed to assess whether the mechanism is relevant to a wider class of oscillatory models.

REFERENCES

1. D.J. Acheson & R. Hide *Hydromagnetics of rotating fluids*, Reports on Progress in Physics, (1973), **36**, p159-221.
2. R.Arlt, R. Hollerbach, G. Rüdiger, A & A, (2003), 401, 1087.
3. J. Aubert, *Steady zonal flows in spherical shell dynamos*, J. Fluid Mech., (2005), **542**, 53-67.
4. S.A. Balbus, *Enhanced Angular Momentum transport in Accretion Disks*, ARR&A, (2003), 41, 555.
5. S.A. Balbus & J.F. Hawley. Astrophys. J. (1991), 376, 214.
6. S.A. Balbus & J.F. Hawley *Instability, turbulence and enhanced transport in accretion disks*, RMP, (1998), 70, 1.
7. S.A. Balbus, J. Bonart, H.N. Latter, and N.O. Weiss, *Differential rotation and convection in the Sun*, Mon. Not. R. Astron. Soc., (2009), 400, 176.
8. A. Brandenburg & K. Subramanian, Phys. Rep., (2005), 417, 1.
9. F.H. Busse, *Convective flows in rapidly rotating spheres and their dynamo action*. (2002), Physics of Fluids, **14**, 1301-1314.
10. F.H. Busse, R.D. Simitev, *Parameter dependences of convection-driven dynamos in rotating spherical fluid shells*, Geophys. Astrophys. Fluid Dyn., (2006), **100**, 341.
11. U.R. Christensen *el al.*, *A numerical dynamo benchmark*. Physics of the Earth and Planetary Interiors, (2001), **128**, 25-34.
12. U.R. Christensen, *Zonal flow driven by strongly supercritical convection in rotating spherical shells*, J. Fluid Mech., (2002), **470**, 115-133.
13. U.R. Christensen, *Dynamo scaling laws and applications to the planets*, Space Sci. Rev., (2009, DOI: 10.1007/s11214-009-9553-2.
14. U.R. Christensen, & J. Aubert, Geophys. J. Int., (2006), **166**, 97.
15. U.R. Christensen, P. Olson. & G.A. Glatzmaier *A dynamo model interpretation of geomagnetic field strucures*, (1998), Geophys. Res. Lett. **25**, 1565-1568.
16. U.R. Christensen, *Geodynamo models: Tools for understanding properties of Earth's magnetic field*, (2011), Physics of the Earth and Planetary Interiors, **187**, 157-169.
17. U.R. Christensen & J. Wicht, *Core Dynamics, Treatise on Geophysics*, Elsevier, Ch. Numerical Dynamo Simulations. (2007).
18. U.R. Christensen, V. Holzwarth and A. Reiners, *Energy flux determines magnetic field strength of planets and stars*, Nature, (2009), **457**, 167.
19. J.-F. Donati, J.D. Landstreet, *Magnetic field of nondegenerate stars*, Annu. Rev. Astro. Astrophys., (2009), 47,333-70.
20. E. Dormy, P. Cardin, D. Jault, *MHD flow in a slightly differentially rotating spherical shell, with conducting inner core, in a dipolar magnetic field*, Earth Planet. Sci. Lett., (1998), **160**, 15-30.
21. D.R. Fearn & M.R.E. Proctor, *Hydromagnetic waves in a differentially rotating sphere*, J. Fluid Mech., (1983a), **128**, 1-20.
22. D.R. Fearn & M.R.E. Proctor *The stabilizing role of differential rotation on hydromagnetic waves*, J. Fluid Mech., (1983b), **128**, 21-36.
23. D.R. Fearn, (1984), Geophys. Astrophys. Fluid Dyn., **30** 227-39.
24. D.R. Fearn, (1985), Geophys. Astrophys. Fluid Dyn., **33** 185-197.
25. D.R. Fearn, (1988), Geophys. Astrophys. Fluid Dyn., **44**, 55-75.
26. D.R. Fearn. *Nonlinear Planetary Dynamos*, in :Lectures on Solar and Planetary Dynamos, (Eds : M.R.E. Proctor and A.D. Gilbert), (1994), CUP, Chapter 7, 219)244.
27. L. Goudard, E. Dormy, *Relations between the dynamo region geometry and the magnetic behavior of stars and planets*, Europhys. Lett., (2008), **83**, 59001.
28. J.F. Hawley, C.F. Gammie, and S.A. Balbus, (1996), Astrophys. J. 464, 690.

29. C.A. Jones, K.M. Kuzanyan, *Compressible convection in the deep atmospheres of giant planets*, Icarus, (2009), 204, 227.
30. Frank J., King A., Raine D., *Accretion Power in Astrophysics*, book, Cambridge University Press,
31. M. Küker, G. Rüdiger, L.L. Kitchatinov, A & A (2011), 530, A48.
32. K. Moffatt, *Magnetic Field Generation in electrically conducting fluids* (1978), book, publication: Cambridge Press.
33. P. Olson, U.R. Christensen, G.A. Glatzmaier, *Numerical modeling of the geodynamo: Mechanisms of field generation and equilibration*, J. Geophys. R., (1999), 104,10383.
34. E.N. Parker, (1955), ApJ, 122, 293.
35. L. Petitdemange, E. Dormy, & S.A. Balbus, *Magnetostrophic MRI in the Earth's Outer Core*, Geophys. Res. Lett., (2008), **35**, L15305, doi:10.1029/2008GL034395.
36. L. Petitdemange, (2010), Geophys. Astro. Fluid Dyn. GAFD, **104**, 287-299, doi:10.1080/03091921003636496.
37. I. Proudman, *The almost-rigid rotation of viscous fluid between concentric spheres*, J. Fluid Mech., (1956), **1**, 505-516.
38. K.-H. Rädler & H. Bräuer, (1987), Astron. Nachr., 308, 101.
39. G. Rüdiger, D. Elstner, & M. Ossendrijver, (2003), A&A, 406, 15.
40. M. Schrinner, K.-H. Rädler, D. Schmitt, M. Rheinhardt, & U.R. Christensen, (2005), Astron. Nachr., 326, 245.
41. M. Schrinner, K.-H. Rädler, D. Schmitt, M. Rheinhardt, & U.R. Christensen, (2007), Geophys. Astrophys. Fluid Dyn., 101, 81.
42. M. Schrinner, L. Petitdemange, E. Dormy, *Oscillatory dynamos and their induction mechanisms*, A&A, (2011), **530**, A140.
43. M. Schrinner, *Global dynamo models from direct numerical simulations and their mean-field counterparts*, A&A, (2011), **533**, A108.
44. J.-A. Sellwood, & S.A. Balbus, *Differential rotation and turbulence in extended HI disks*, ApJ., (1999), **511**, 660-665.
45. K. Stewartson, *On almost rigid rotations*, Part 2, J. Fluid Mech., (1966), **26**, 131-144.
46. E. Takahashi, A. Matsushima & Y. Honkura, *Simulation of a quasi-Taylor state magnetic field including polarity reversals on the Earth Simulator*. Science, (2005), **309**, 459-461.
47. J.B. Taylor, *The Magneto-hydrodynamics of a rotating fluid and the Earth's dynamo problem*, Proc. R. Soc., (1964), **274**, 274-283.
48. J.E. Vidale, D.A. Dodge, and P.S. Earle, *Slow differential rotation of the Earth's inner core indicated by temporal changes in scattering*, Nature, (2000), **405**, 445-448.