FOURIER PHASE ANALYSIS IN RADIO-INTERFEROMETRY

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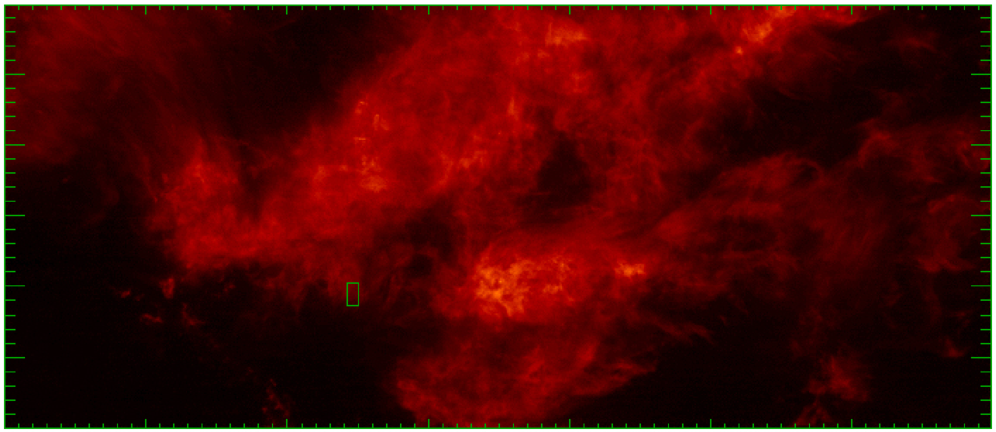
NAOC Seminar, Beijing

September 22, 2006

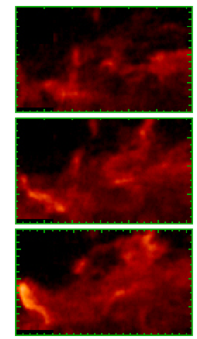
Complex structures in density and velocity

TAURUS



(Goldsmith et al., in prep.)

POLARIS



(Hily-Blant et al., in prep.)

- Dynamical structures on scales 0.01 pc to 100 pc

Research topics

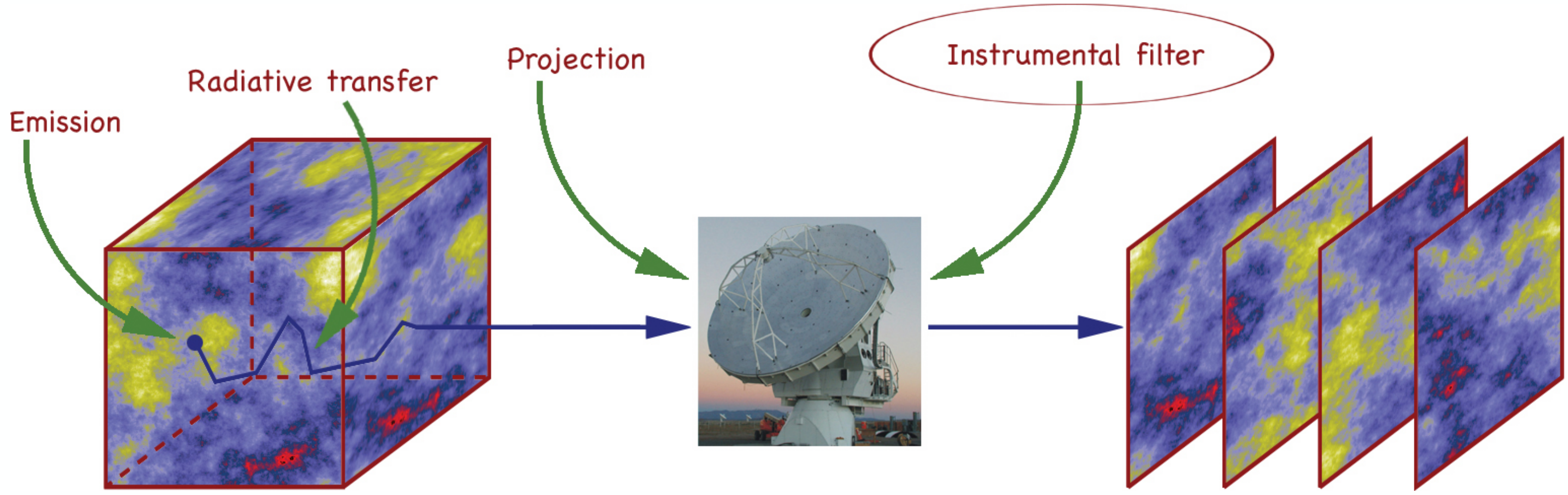
- Turbulent support versus star formation
- CMB foreground

A new generation of observational means

- High angular and spectral resolutions
- Wide dynamical and frequency ranges covered

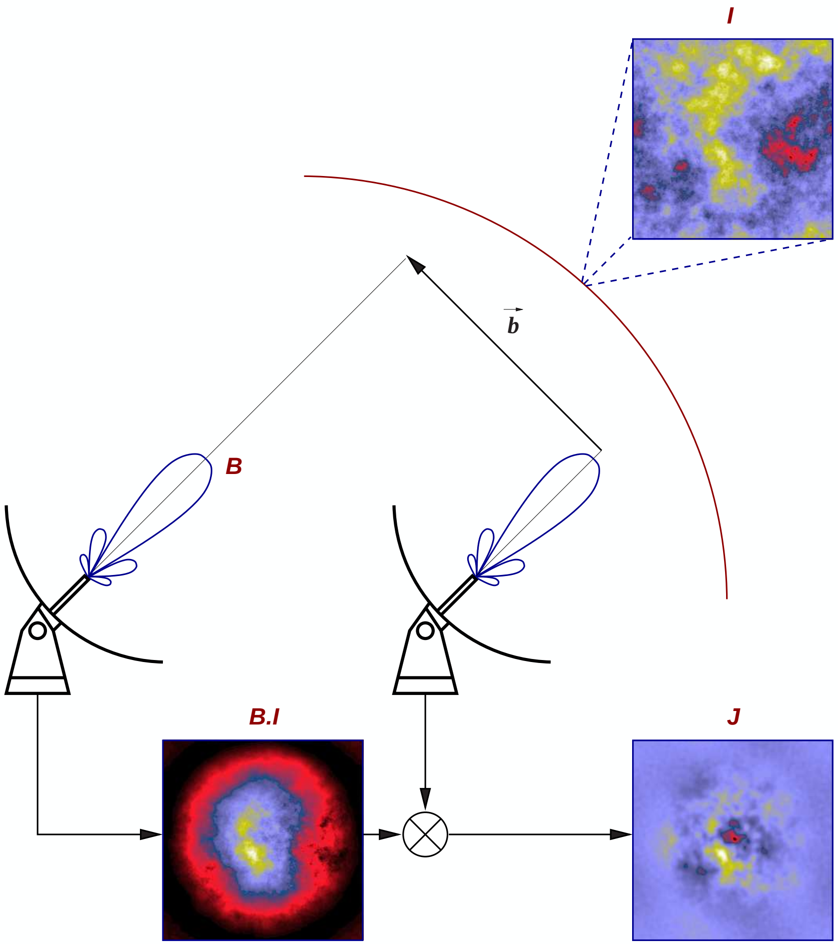
Makes inversion a possibility

The inversion problem



- **Physical fields (3D) are projected onto observables (2D + 1D)**
- **Recovering information on ρ, $\vec{v}$, etc... from the channel maps is a vast problem**
- **Requires understanding the effects of instrumental filters**

How does the instrumental filter alter structure ?



Antenna pairs measure correlations at lag $\vec{b}$

$$J = T_F^{-1}[C.T_F[B.I]] = T_F^{-1}[V]$$

$C(\vec{b})$: (u, v) cover

$V(\vec{b})$: visibility function

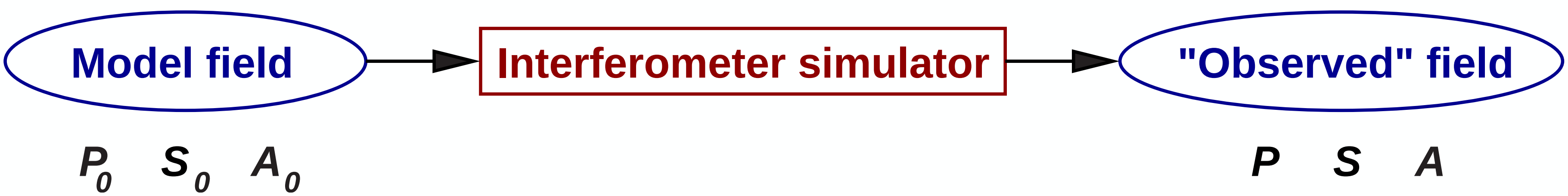
How is I's structure encoded in J ?

Usual structure characterization tools

Statistical measures on an n-dimensional field F

- *Second order structure function* $S(\vec{r}) = \left\langle \left[F(\vec{x}+\vec{r}) - F(\vec{x})\right]^2 \right\rangle_{\vec{x}}$
- *Autocorrelation function* $A(\vec{r}) = \langle F(\vec{x}) F(\vec{x}+\vec{r}) \rangle_{\vec{x}}$
- *Power spectrum* $P(\vec{k}) = |\widehat{F}(\vec{k})|^2$

Direct numerical approach



What statistical tools are the most reliable ?

Fractional Brownian motions

Simple statistical behavior

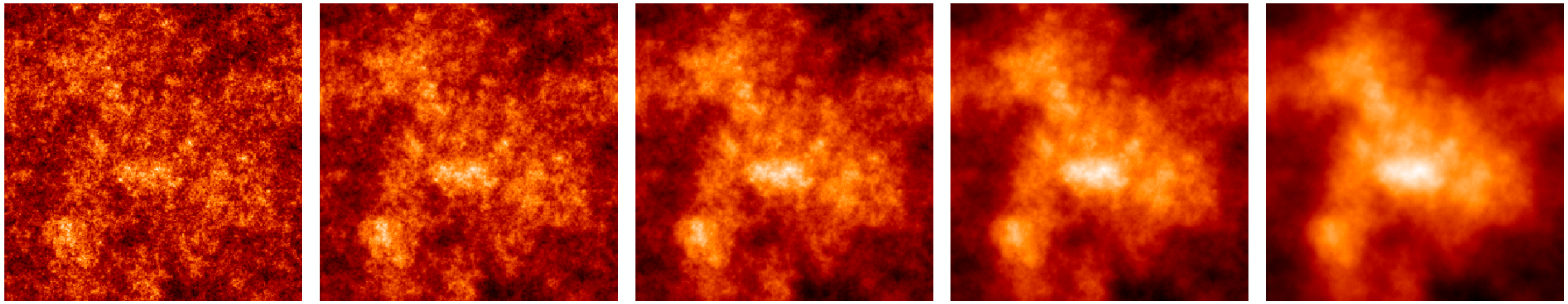
- $S(\vec{r}) \propto |\vec{r}|^{2H}$ with $H \in [0, 1]$
- $P(\vec{k}) \propto |\vec{k}|^{-\beta}$ with $\beta = 2H + n$
- Fully random phases

Numerical implementation

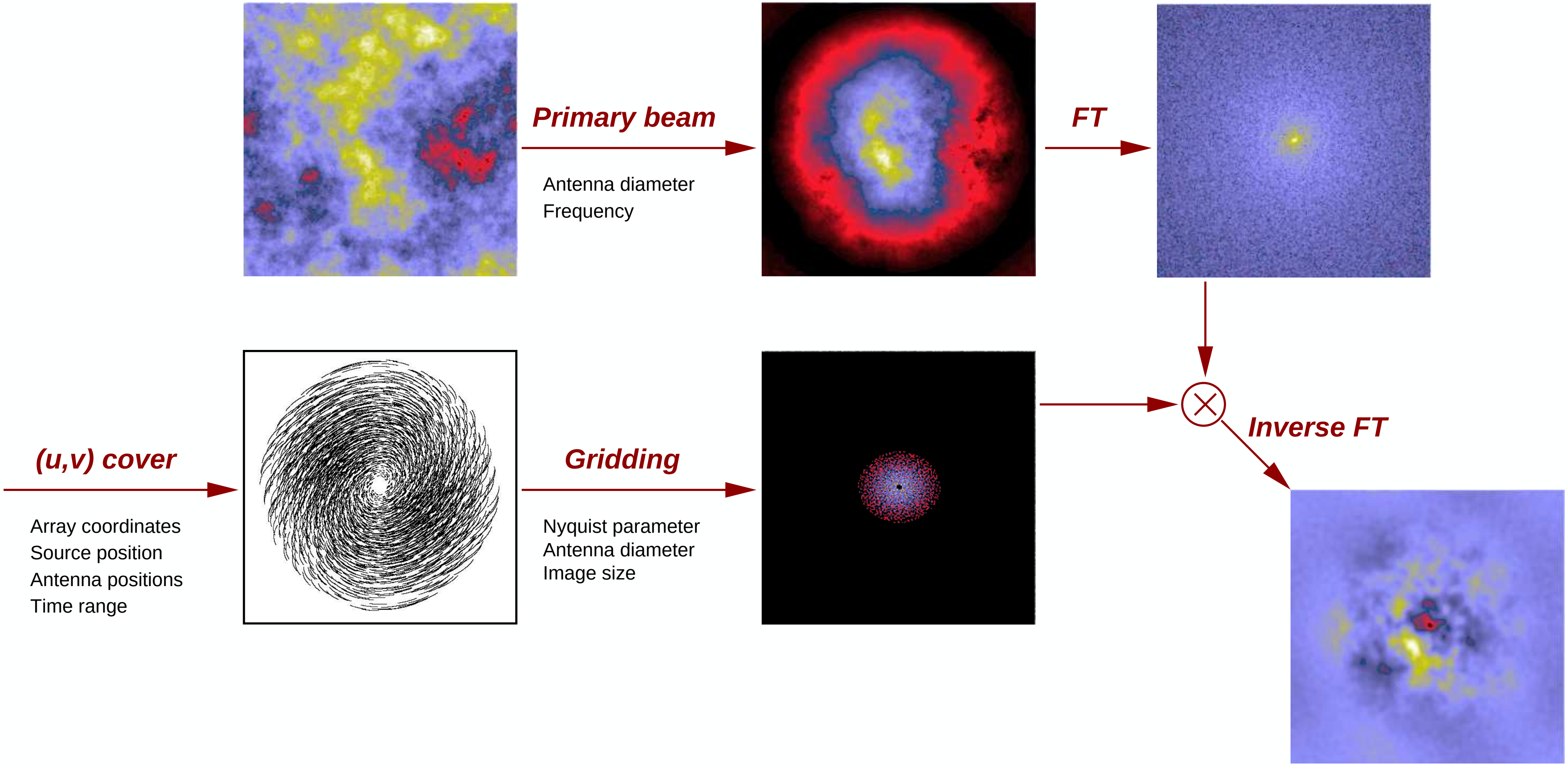
- Ease of generation in Fourier space
- Models of the diffuse interstellar medium

(Stutzki et al., 1998; Bensch et al., 2001; Brunt & Heyer, 2002; Miville-Deschênes et al., 2003; Levrier, 2004)

$\beta = 2$ $\beta = 2,5$ $\beta = 3$ $\beta = 3,5$ $\beta = 4$



Interferometer simulator



- Visibility based $\Longrightarrow$ Possibility to include noise
- Homogeneous arrays only / Flexible configurations

Simulation parameters

Simulated instruments

- Atacama Large Millimeter Array
- Very Large Array
- IRAM Plateau de Bure Interferometer

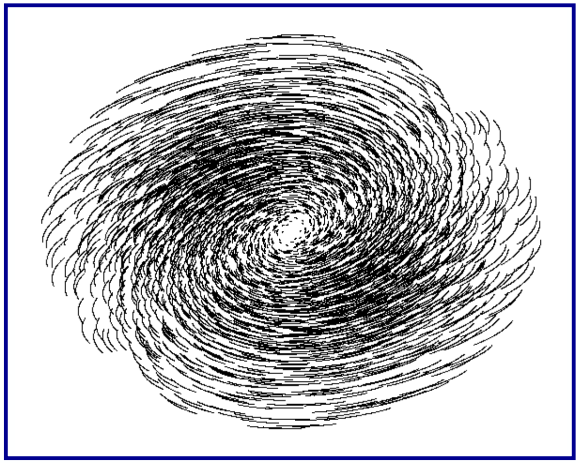
Observing parameters

- Array location : longitude -67.75°, latitude -23.02° $\Longleftrightarrow$ ALMA
- Source declination : $\delta = -20^\circ$
- Observing wavelength : $\lambda = 1.3$ mm
- Source is tracked as long as it remains at least 10° above the horizon

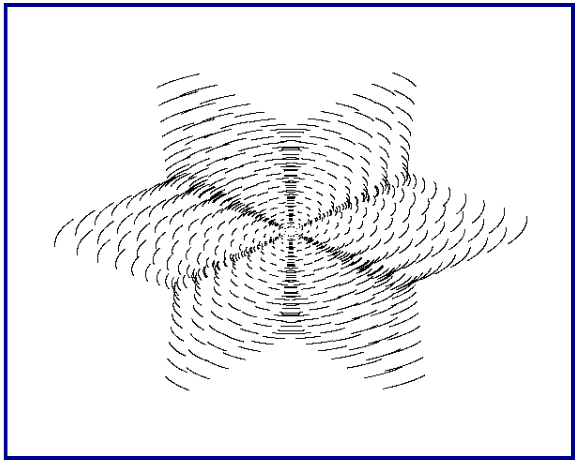
- 64 antennae with 12 meter diameter
- Frequency range: 30 GHz - 950 GHZ
- 4096 spectral channels
- 16 GHz bandwidth
- Baselines: 150 m - 18 km
- First antennae: 2007 / Full array: 2012

Over 4000 instantaneous baselines $\Longrightarrow$ Excellent (u, v) coverage

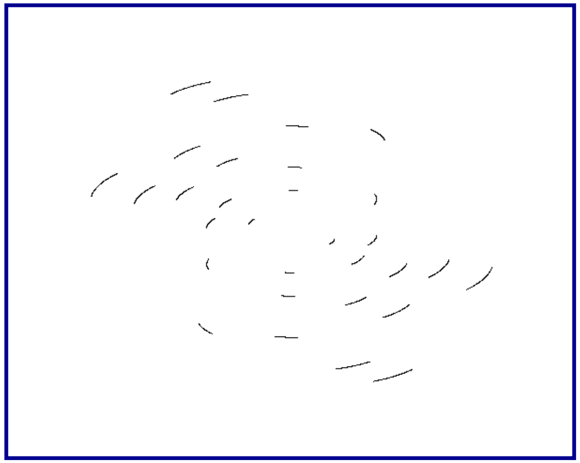
Atacama Large Millimeter Array



Very Large Array

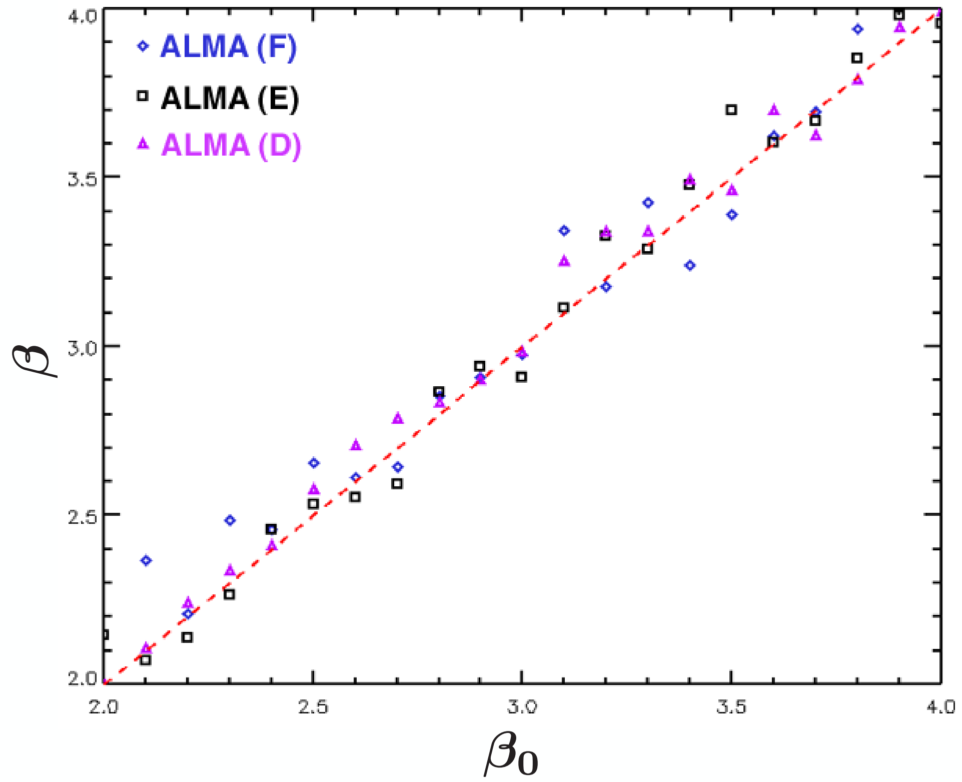


Plateau de Bure Interferometer

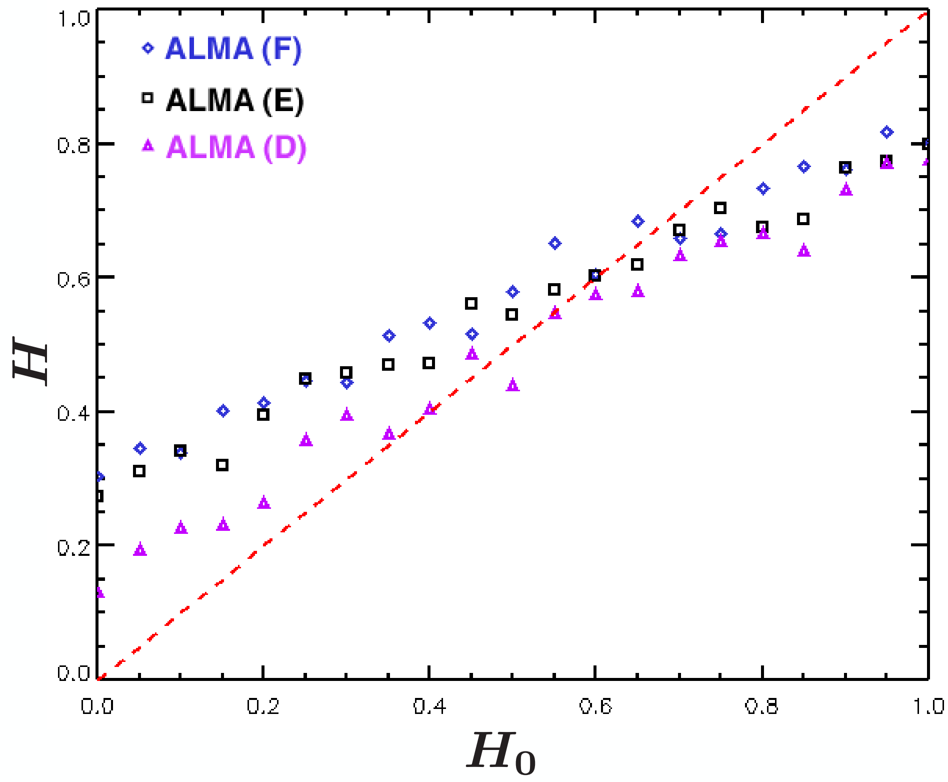


Power spectrum and structure function stability

Power Spectrum



Structure function



Power spectrum is more stable

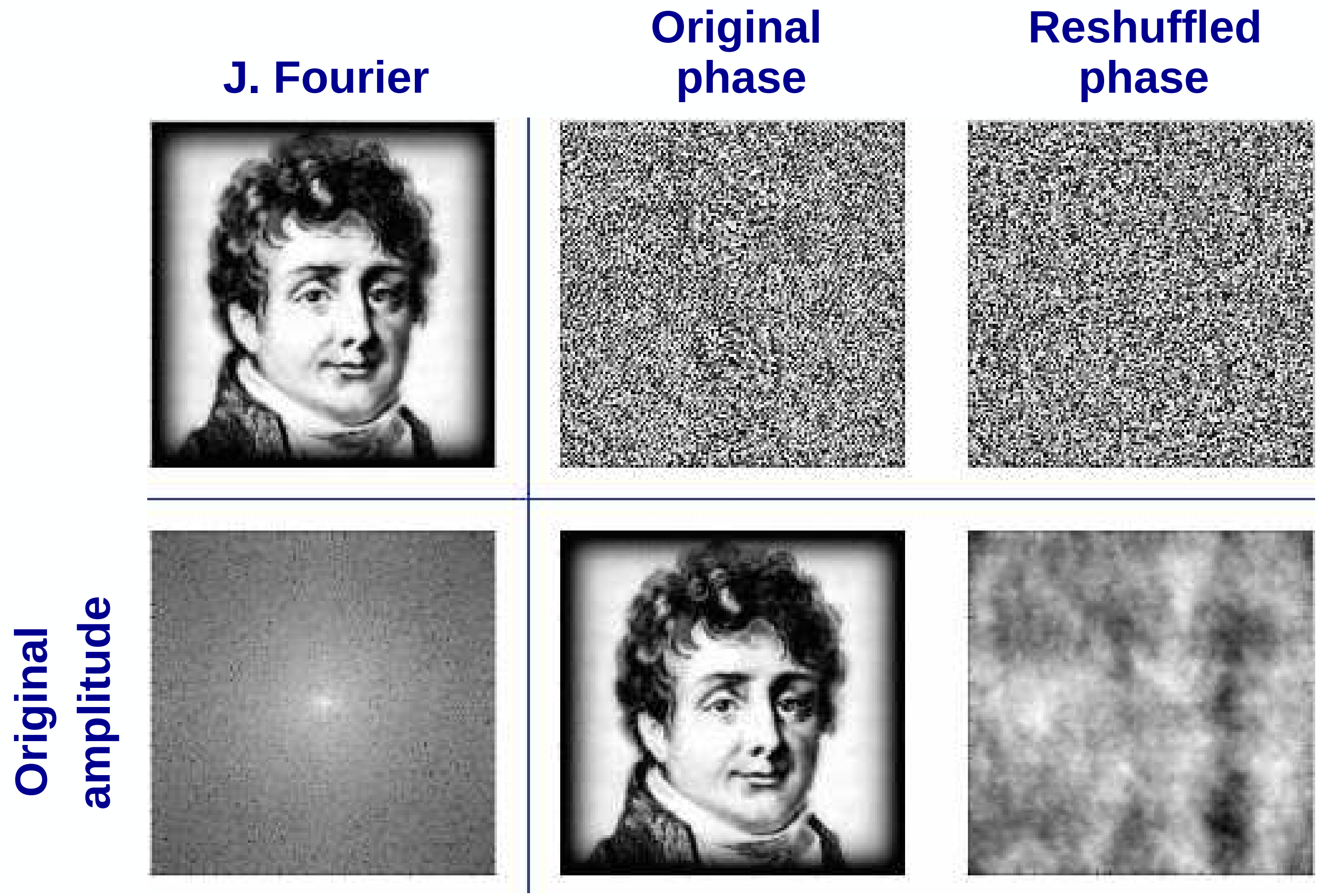
Adaptation of statistical tools to the measurement space

- Structure function $\Leftrightarrow$ *Direct space* $\Leftrightarrow$ Single dish
- Power spectrum $\Leftrightarrow$ *Fourier space* $\Leftrightarrow$ Aperture synthesis

Going further...

- Visibilities are amplitude + phase
- Power spectra only make use of amplitudes

How to make use of the phases ?

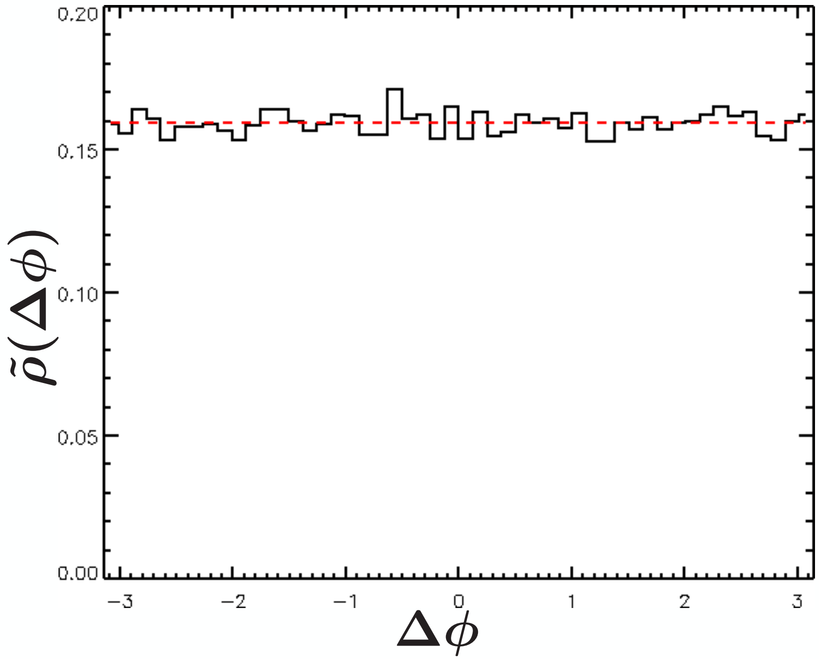


Information in the Fourier-spatial distribution of phases

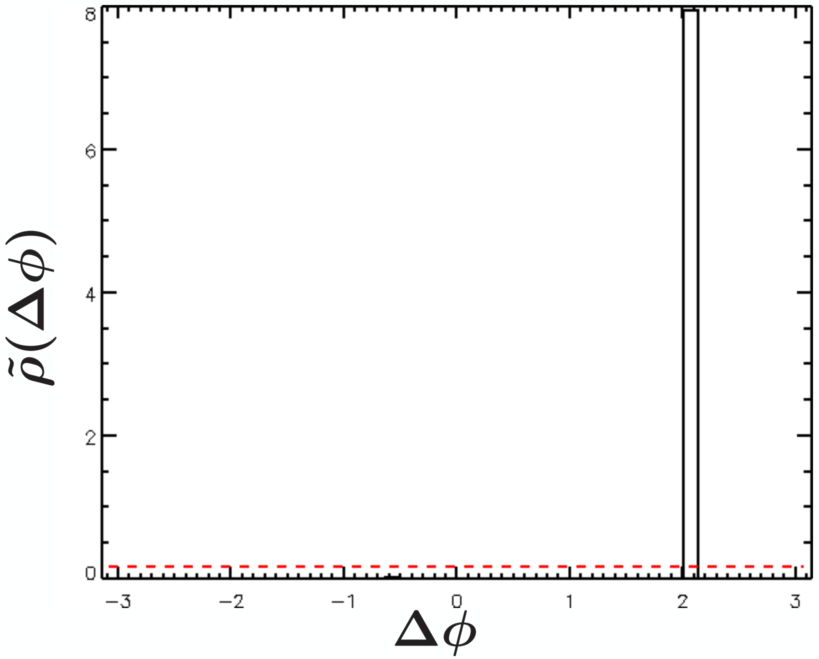
- Defined as $\Delta\phi(\vec{k},\vec{\delta}) = \phi(\vec{k}+\vec{\delta}) - \phi(\vec{k})$ for a given lag vector $\vec{\delta}$
- Statistics of phase increments should trace the structure lost in the reshuffling
- Probability distribution functions $\rho(\Delta\phi)$ approximated by histograms $\tilde{\rho}(\Delta\phi)$

Limiting cases

Fractional Brownian motion

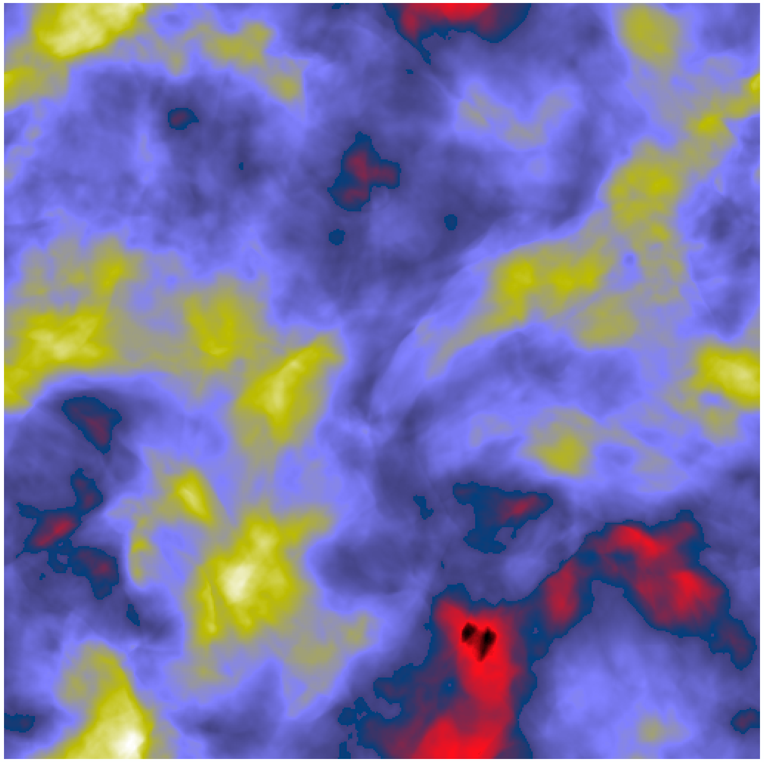


Single point source

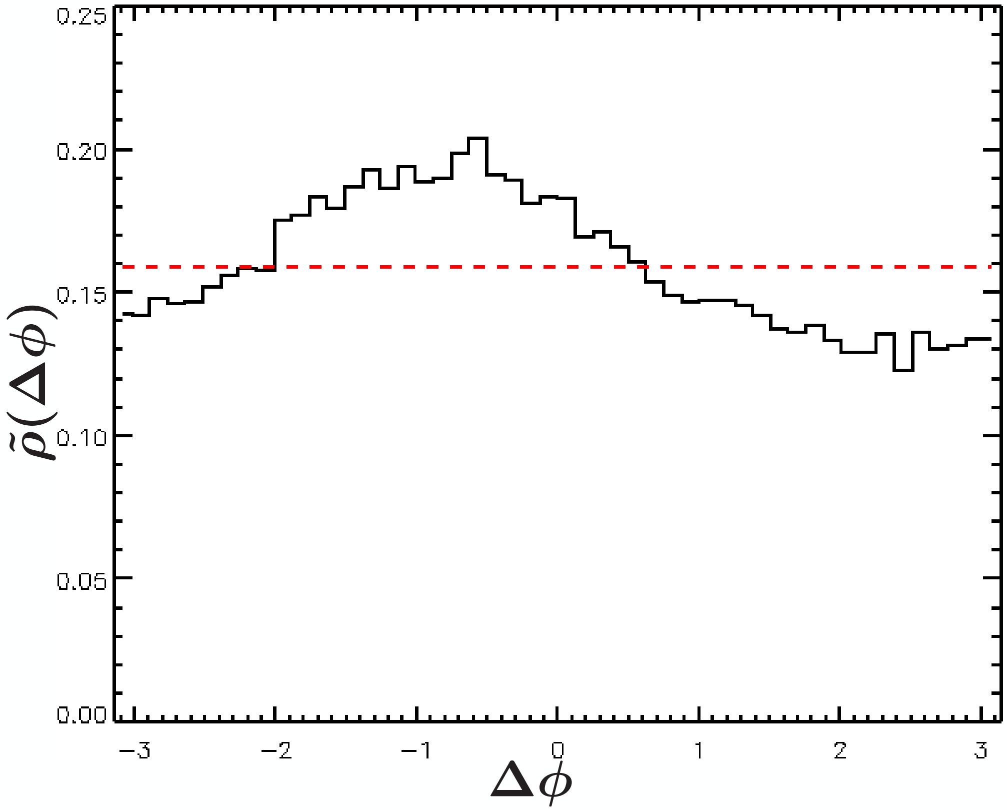


Compressible hydrodynamical turbulence simulation *(Porter et al., 1994)*

512^2 Column density



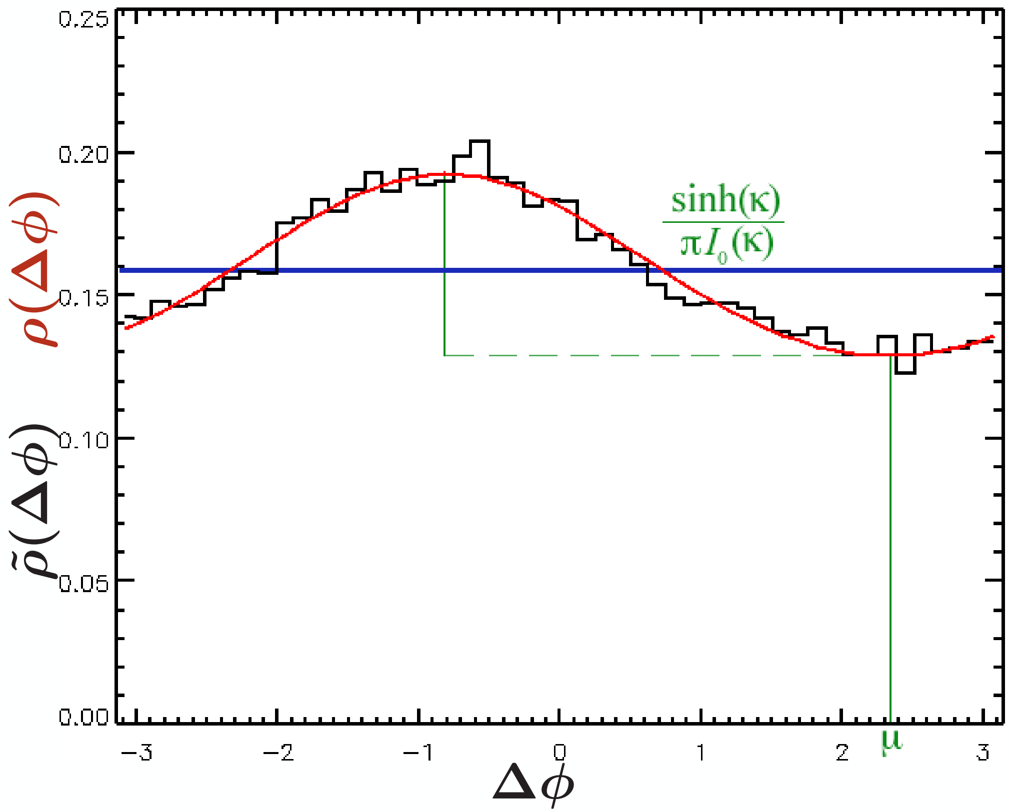
$\vec{\delta} = \vec{e}_x$



Requires quantification of non-uniformity

$$\rho(\Delta\phi) = \frac{1}{2\pi I_0(\kappa)} e^{-\kappa cos(\Delta\phi - \mu)}$$

(Watts et al., 2003)

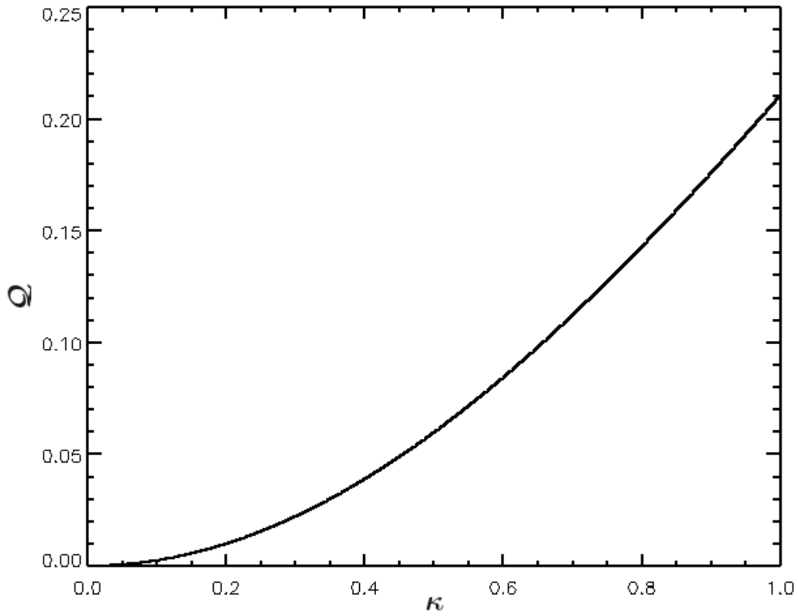


Phase entropy and structure quantity

Characterizations of non-uniformity

- **Phase entropy :** $\mathcal{S}(\vec{\delta}) = -\int_{-\pi}^{\pi} \rho(\Delta\phi)\, ln[\rho(\Delta\phi)]d\Delta\phi$ *(Polygiannakis & Moussas, 1995)*
- **"Phase structure quantity" :** $\mathcal{Q}(\vec{\delta}) = ln\,(2\pi) - \mathcal{S}(\vec{\delta}) \geq 0$

Relation to the von Mises parameter κ



$$\mathcal{Q} = \kappa\frac{I_1(\kappa)}{I_0(\kappa)} - ln\,[I_0(\kappa)]$$

The trouble with estimators

Finite size images

- Uniform PDFs do not lead to uniform histograms
- Structure quantities $\tilde{Q}$ for numerical fractional Brownian motions are not zero
- May lead to false detection of phase structure

What is the contribution of statistical noise to $\tilde{Q}$?

Parameters

- Number of phase increments : p
- Number of histogram bins : n

Given these, estimate an upper limit of $P(\tilde{Q} > x)$ for any $x > 0$

Suppose uniform distribution $\rho(\Delta\phi)$

Extraordinary histograms

- One value $\tilde{\rho}_i$ strays "too much" from the uniform value r (quantified by $\epsilon > 0$)
- This defines the event $\Omega_\epsilon = \{\exists i; |\tilde{\rho}_i - r| > \epsilon r\}$
- For large enough p and n, the central limit theorem applies :

$$P(\Omega_\epsilon) \leq P_1 = n - n\mathrm{Erf}\left(\epsilon\sqrt{\frac{p}{2(n-1)}}\right)$$

Regular histograms

- Results due to Castellan (2000) http://www.math.u-psud.fr/theses-orsay/2000/6039.html

$$P(\tilde{Q} > x) \leq P_2 = P\left(\chi^2 > \frac{2(1-\epsilon)^2 p x}{1+\epsilon}\right)$$

General case

$$P(\tilde{Q} > x) \leq P_1 + P_2$$

Adaptive and fixed upper limits

Adaptive procedure

- 1 : Choose value of P_1 such that $P_1 \ll 1$
- 2 : Deduce ϵ given n and p
- 3 : Choose value of P_2 such that $P_1 \ll P_2 \ll 1$
- 4 : Use quantiles of χ^2 to deduce x

$\tilde{Q}$ is less than x with probability $1 - P_2$

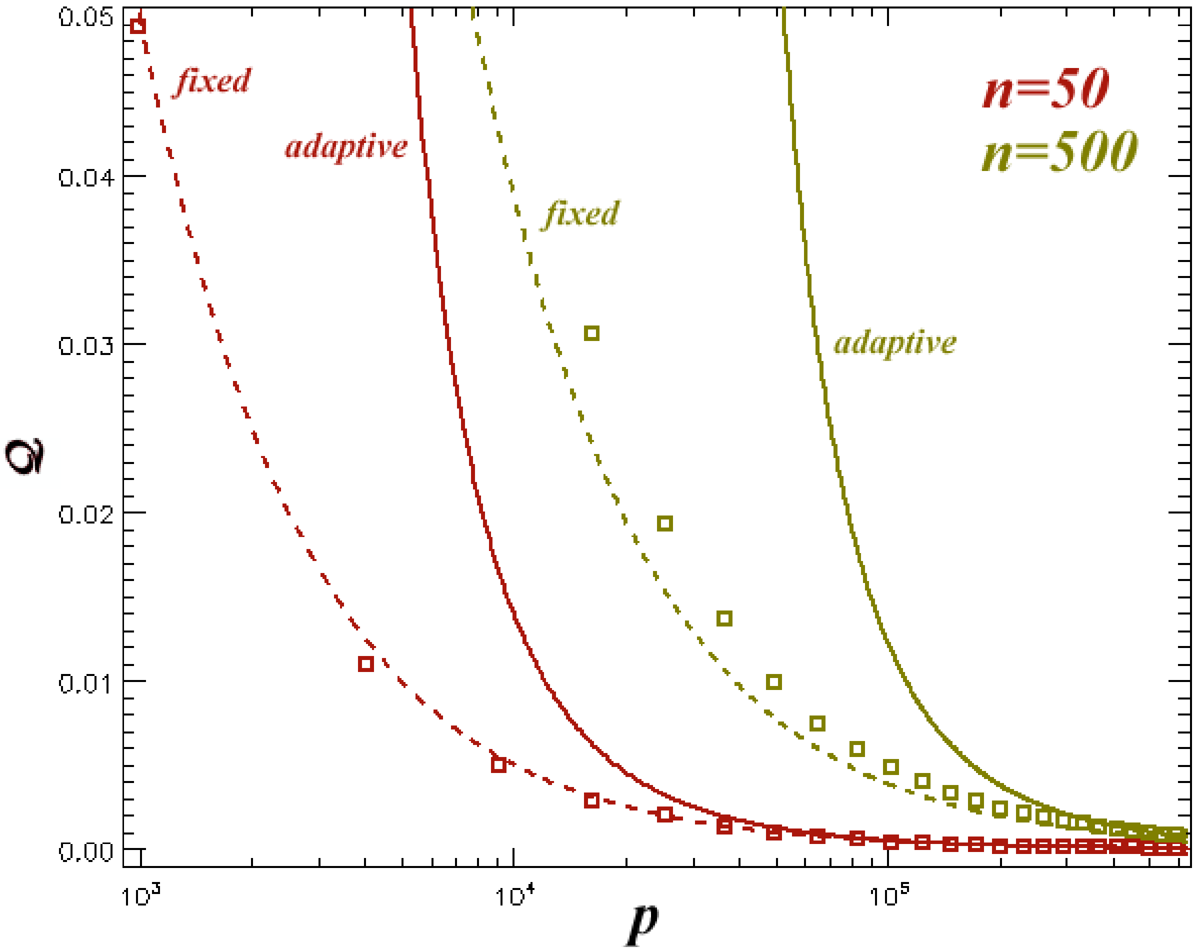
Fixed procedure

- Steps 1 and 2 replaced by fixing a value for ϵ
- Steps 3 and 4 as before

$\tilde{Q}$ is less than x for "usual" n and p but P_1 may be greater than 1

Adaptive procedure may be too conservative
Fixed procedure may fail

- Generate Fractional Brownian motions and compute $\tilde{Q}$
- Vary field size and number of bins



Fourier phases and interferometry

Primary beam attenuation

- Convolution in Fourier space
- Mosaic observations effectively reduce kernel size

Not considered $\Longleftrightarrow$ Pointlike antennae

Pillbox gridding

- Measured phases associated with "wrong" wavenumber
- Model brightness distributions already gridded

Not considered $\Longleftrightarrow$ Phase constant over each pixel

Atmospheric phase noise

- Atmospheric turbulence makes phase space- and time-dependent

Considered ... See later

Phase structure quantity in observations

The input brightness distribution

Column density of a compressible hydrodynamical simulation for which $\mathcal{Q} \simeq 0.01$

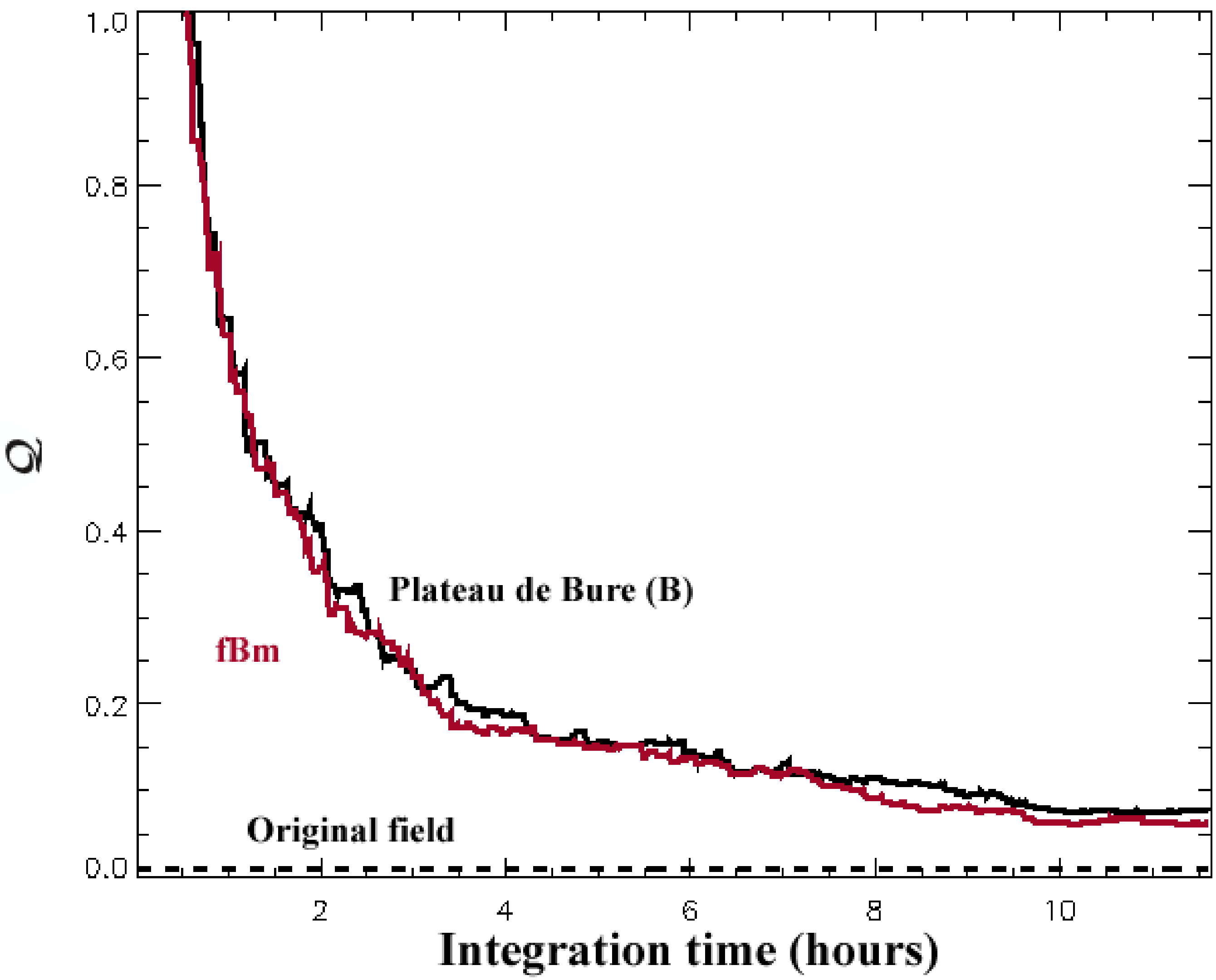
The parameters

- 3 possible arrays
- Single or multiple configurations (possibly trimmed)
- Atmospheric phase noise

The questions asked

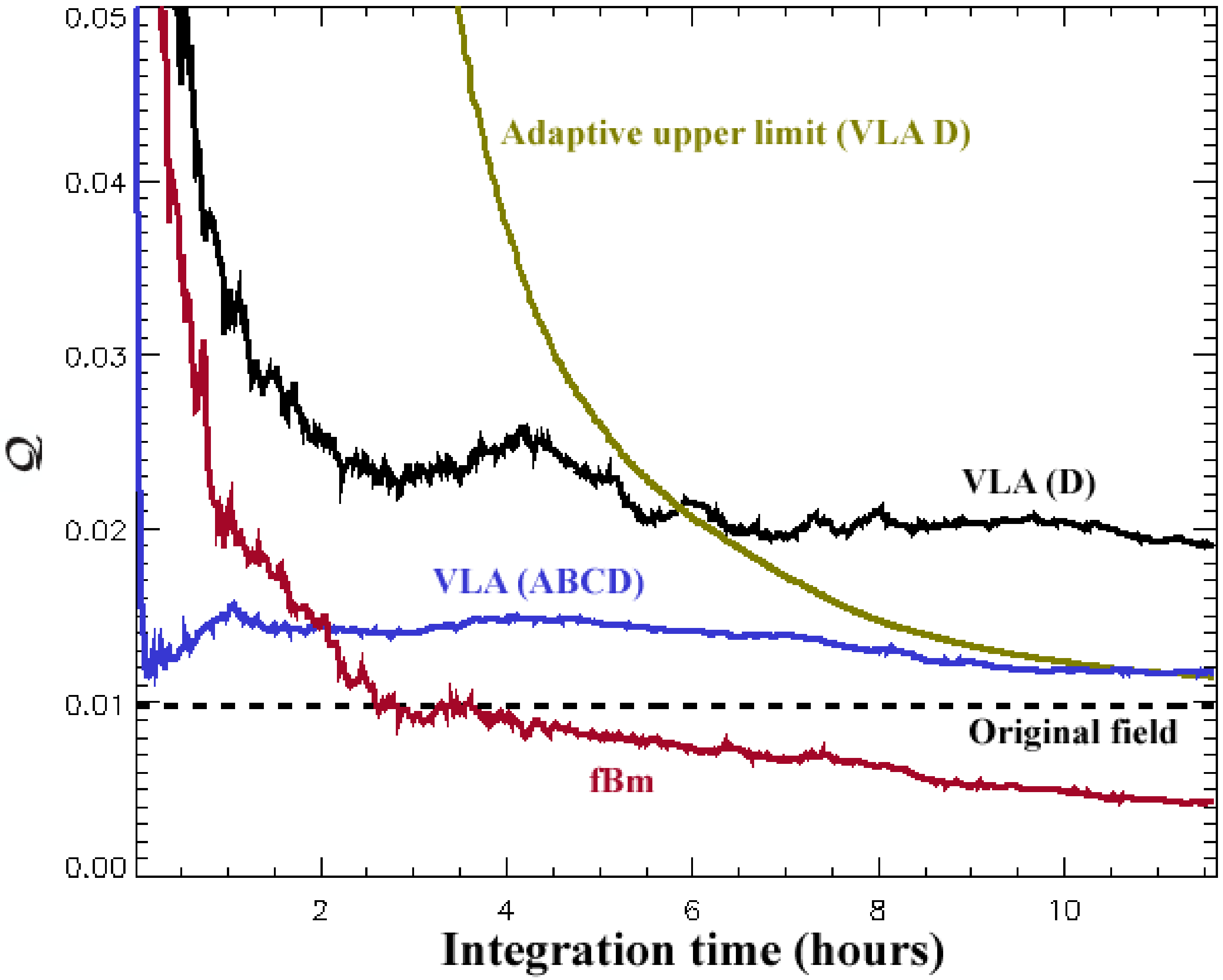
- How long does it take to achieve a significant detection of phase structure ?
- How long does it take to recover the actual phase structure quantity ?
- What level of atmospheric turbulence still allows detection of phase structure ?

Noise-free observations with Plateau de Bure



Detection not possible

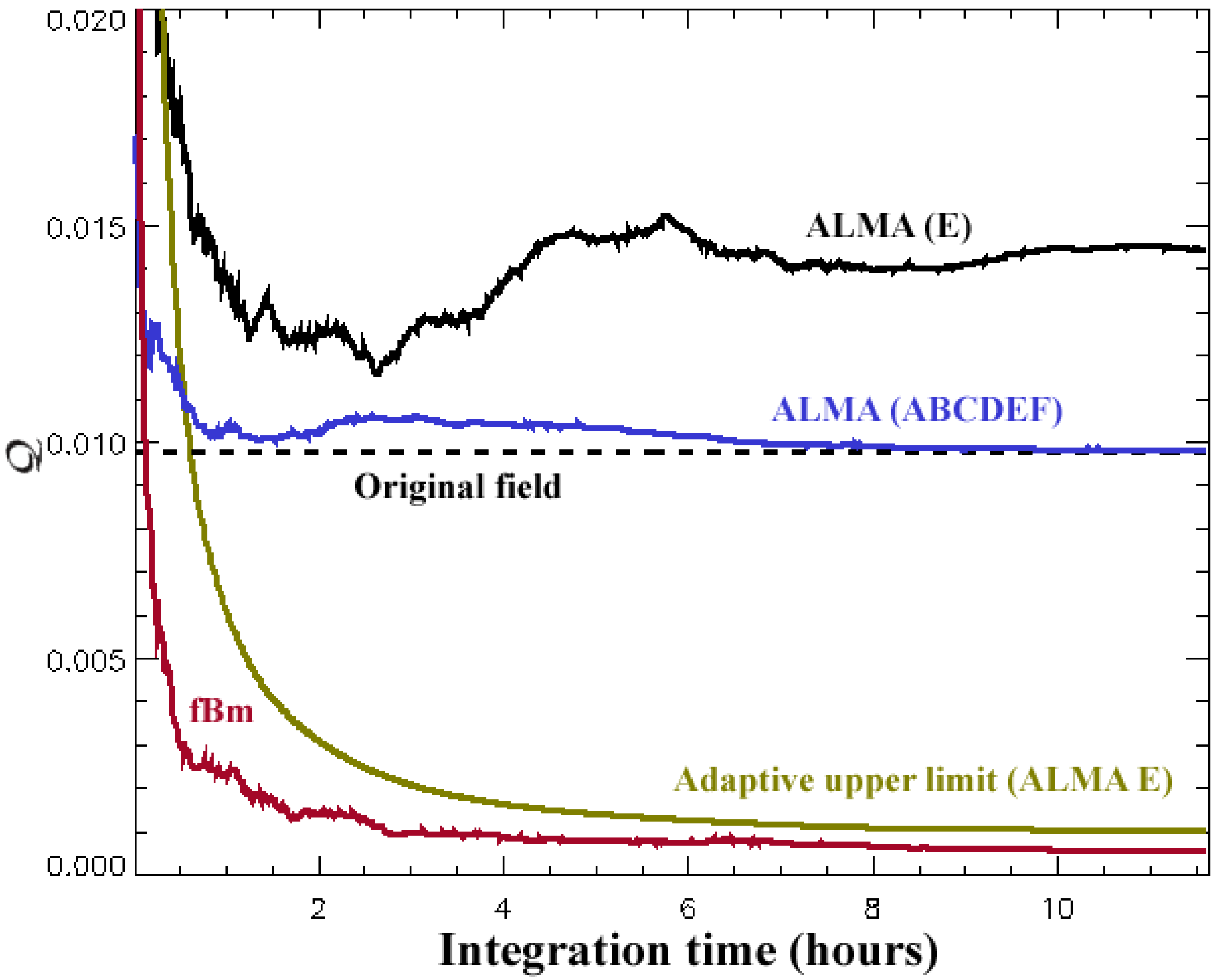
Noise-free observations with the VLA



Detection possible with single configuration

Measurement not possible with multiple configurations

Noise-free observations with ALMA

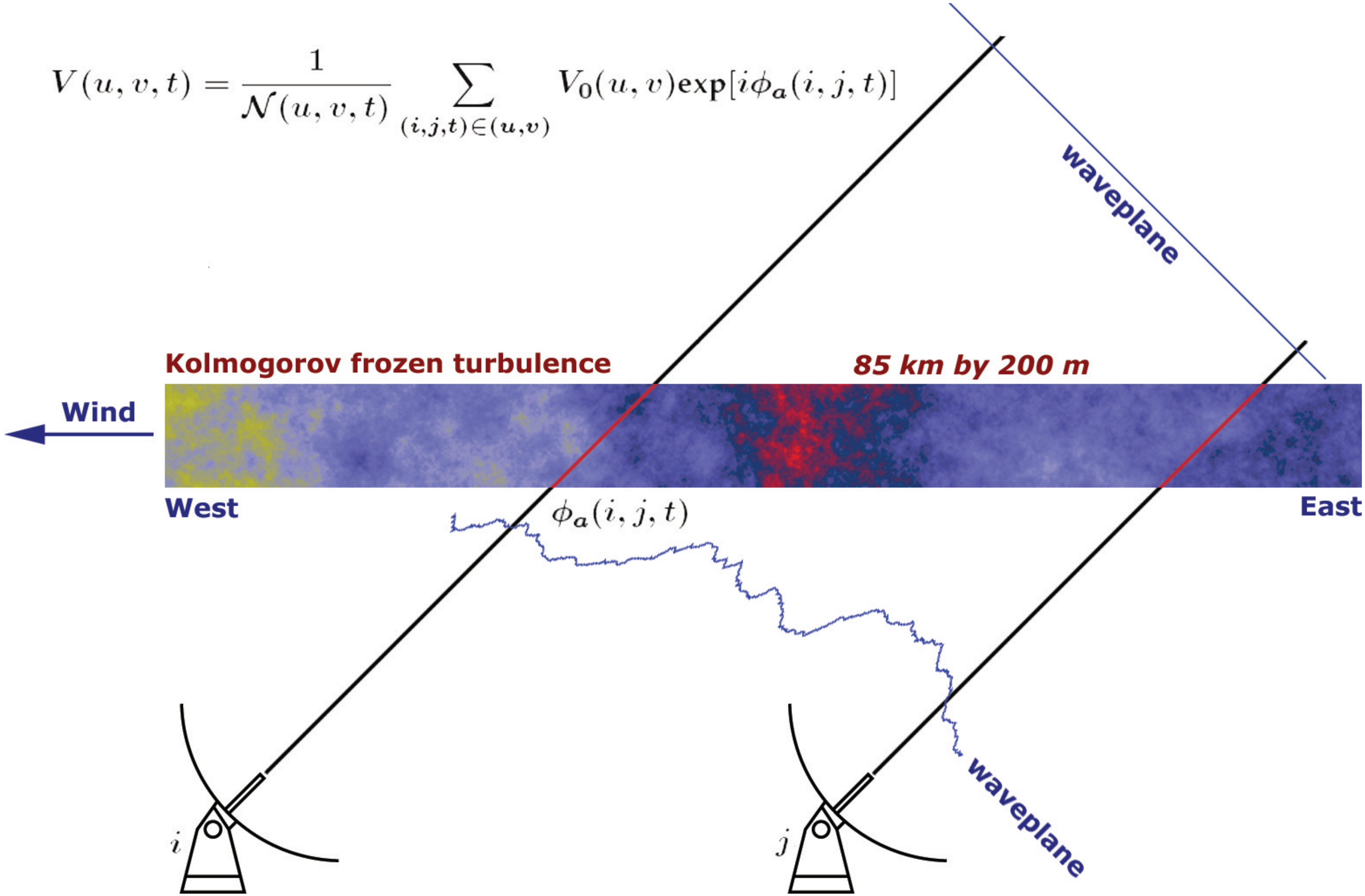


Detection possible with single configuration

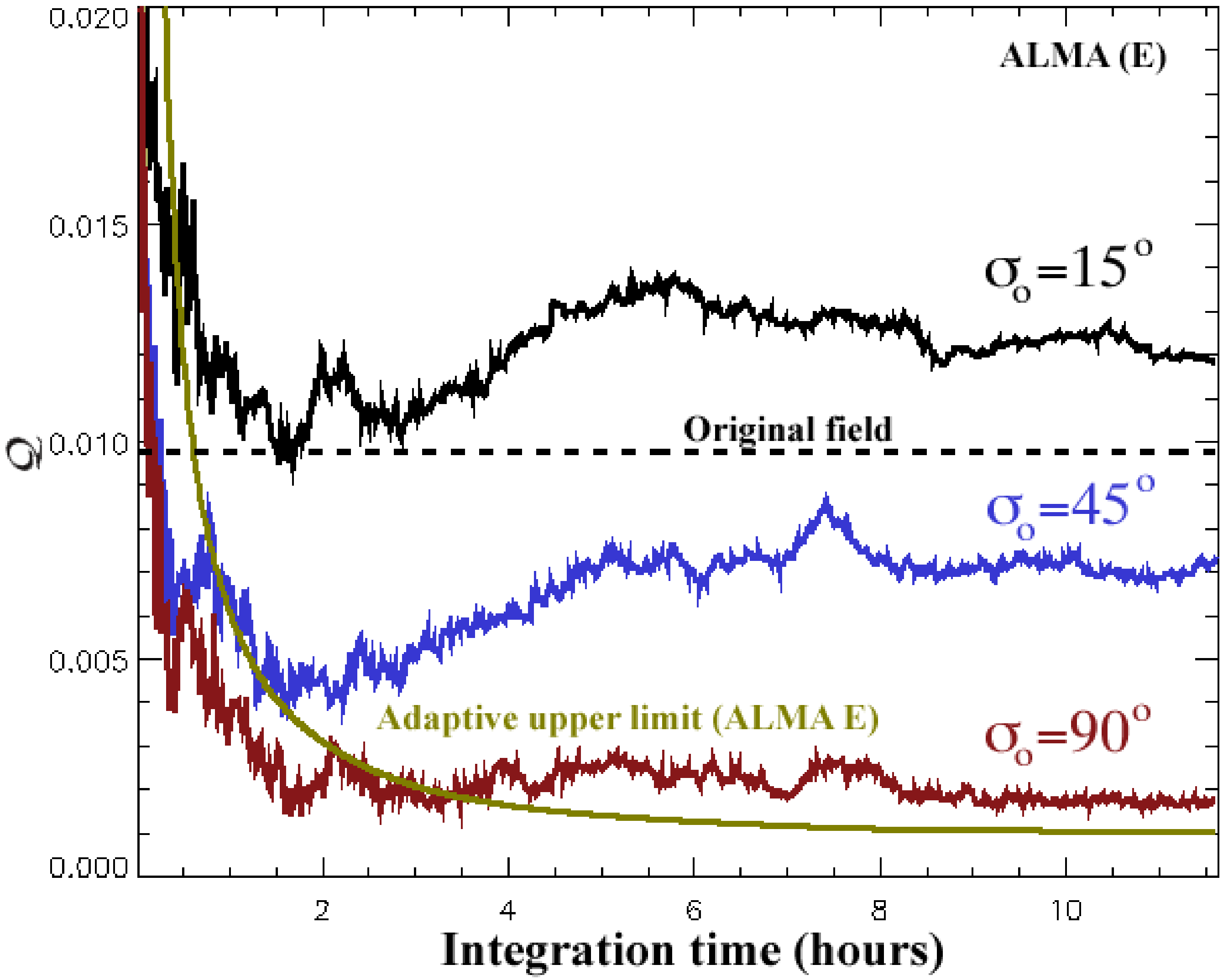
Measurement possible with multiple configurations

Atmospheric phase noise

$$V(u,v,t) = \frac{1}{\mathcal{N}(u,v,t)} \sum_{(i,j,t)\in(u,v)} V_0(u,v)\exp[i\phi_a(i,j,t)]$$



Noisy observations with ALMA



rms phase delay σ_0 :

- 100m baseline
- 1.3 mm wavelength
- Zenith observation

Chajnantor: 15° to 60°

Detection possible with single configuration

Detection of phase structure

- Requires extended ALMA configuration
- Atmospheric phase noise not critical

Measurement of phase structure

- Requires multiple ALMA configurations

Open questions

- Allow for variations of $\vec{\delta}$
- Interpretation of phase structure quantities $\Longleftrightarrow$ Physical processes