UTILISATION DES PHASES POUR L'ANALYSE DE LA COHÉRENCE EN INTERFÉROMÉTRIE RADIO

François Levrier

Journée Plasmas, Jussieu

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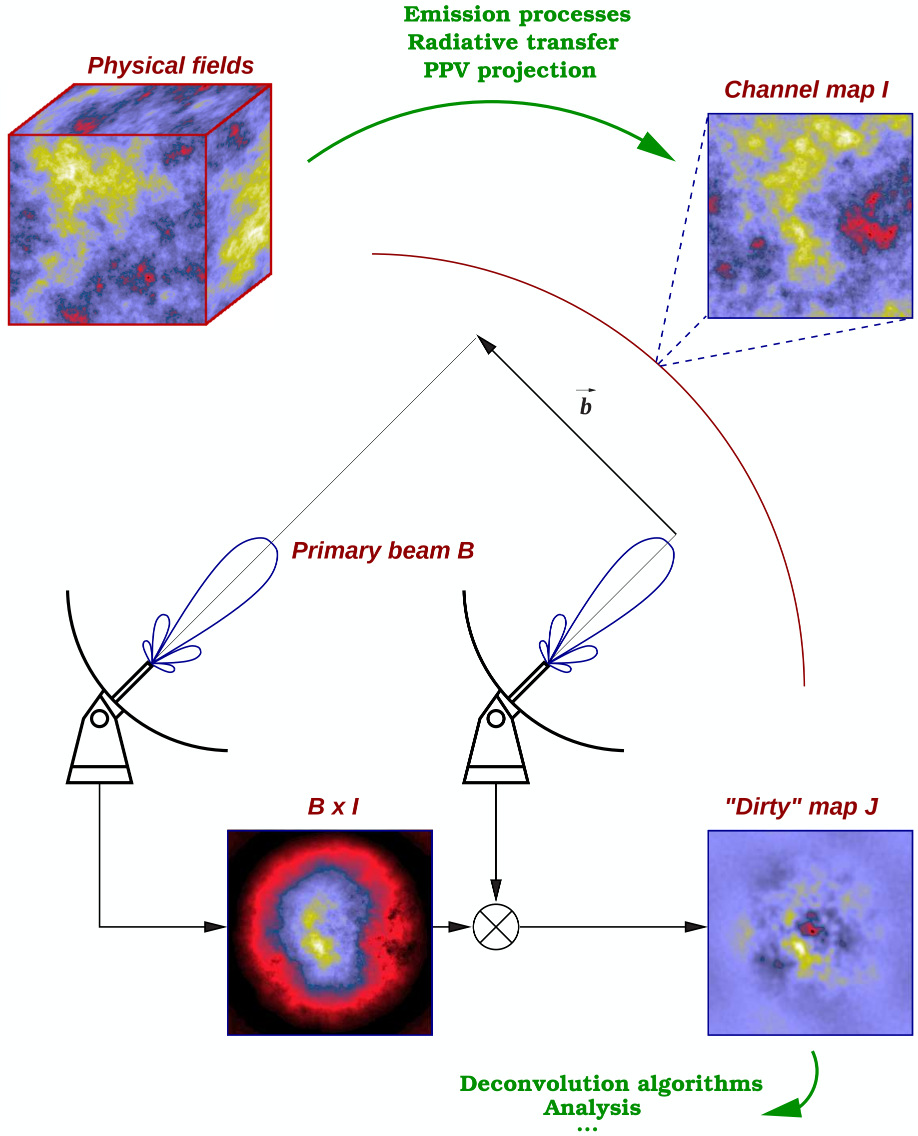


- Projection on a PPV hybrid space
- Antenna pairs measure correlations
- Primary beam attenuation : B
- Incomplete sampling : C

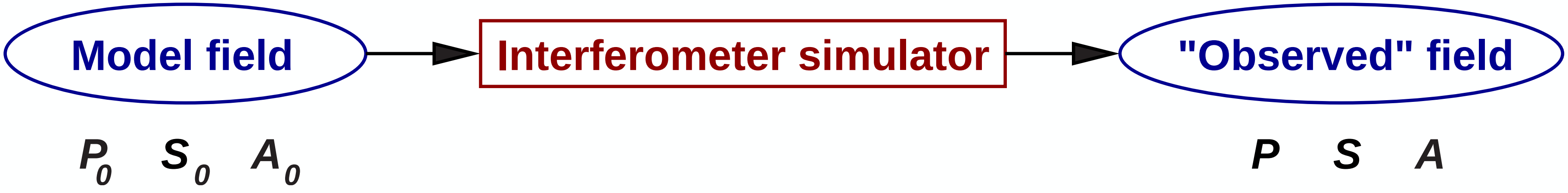
$$J = T_F^{-1}[C.T_F[B.I]] = T_F^{-1}[V]$$

How does all this alter structure ?

Statistical measures on an n-dimensional field F

- *Second order structure function* $S(\vec{r}) = \left\langle [F(\vec{x}+\vec{r}) - F(\vec{x})]^2 \right\rangle_{\vec{x}}$
- *Autocorrelation function* $A(\vec{r}) = \langle F(\vec{x})F(\vec{x}+\vec{r})\rangle_{\vec{x}}$
- *Power spectrum* $P(\vec{k}) = |\widehat{F}(\vec{k})|^2$

Direct numerical approach



What statistical tools are the most reliable ?

Fractional Brownian motions

Simple statistical behavior

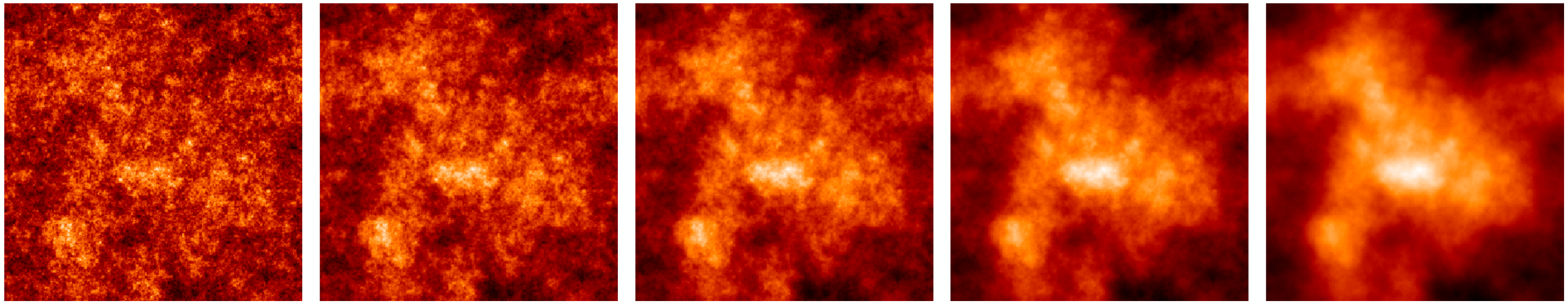
- $S(\vec{r}) \propto |\vec{r}|^{2H}$ with $H \in [0, 1]$
- $P(\vec{k}) \propto |\vec{k}|^{-\beta}$ with $\beta = 2H + n$
- Fully random phases

Numerical implementation

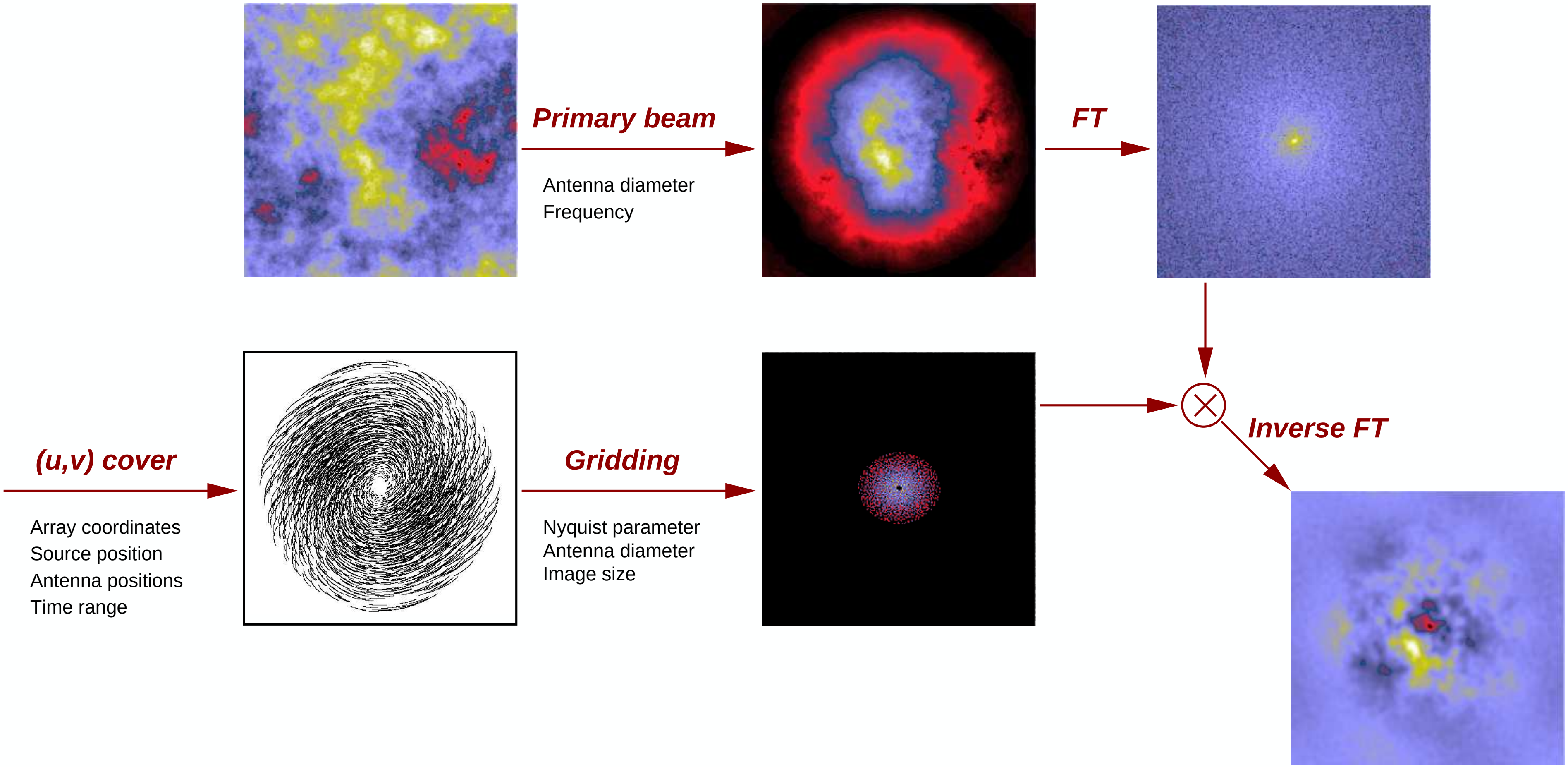
- Ease of generation in Fourier space
- Models of the diffuse interstellar medium

(Stutzki et al., 1998; Bensch et al., 2001; Brunt & Heyer, 2002; Miville-Deschênes et al., 2003; Levrier, 2004)

$\beta = 2$ $\beta = 2,5$ $\beta = 3$ $\beta = 3,5$ $\beta = 4$

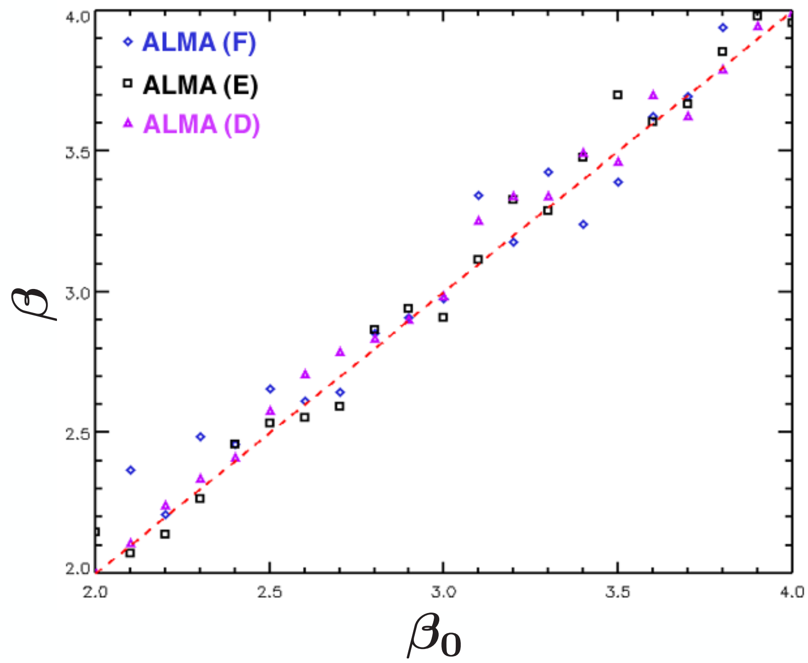


Interferometer simulator

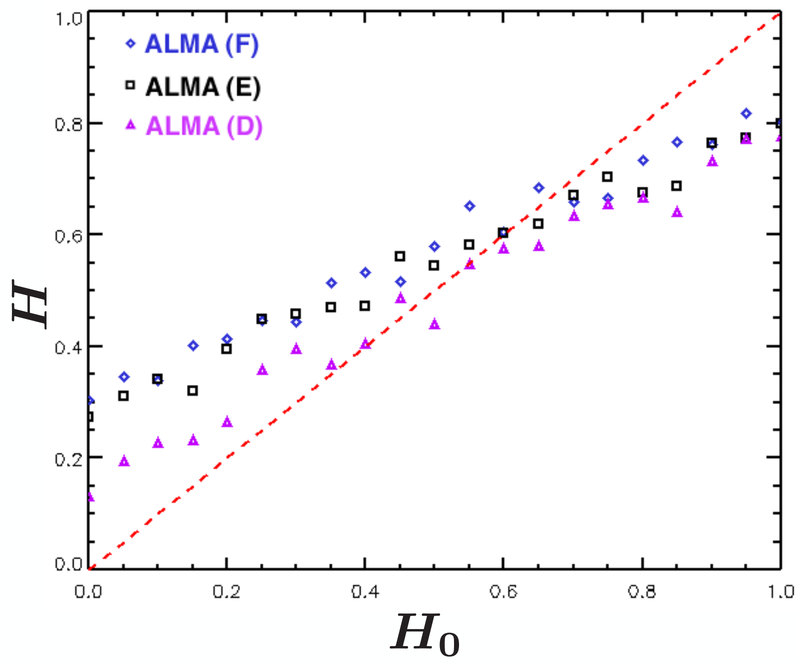


- **Interferometer simulator written in IDL (my own private MeqTrees)**
- **Visibility based / Homogeneous arrays / Flexible configurations**
- **ALMA / VLA / PdBI / ...**

Power Spectrum



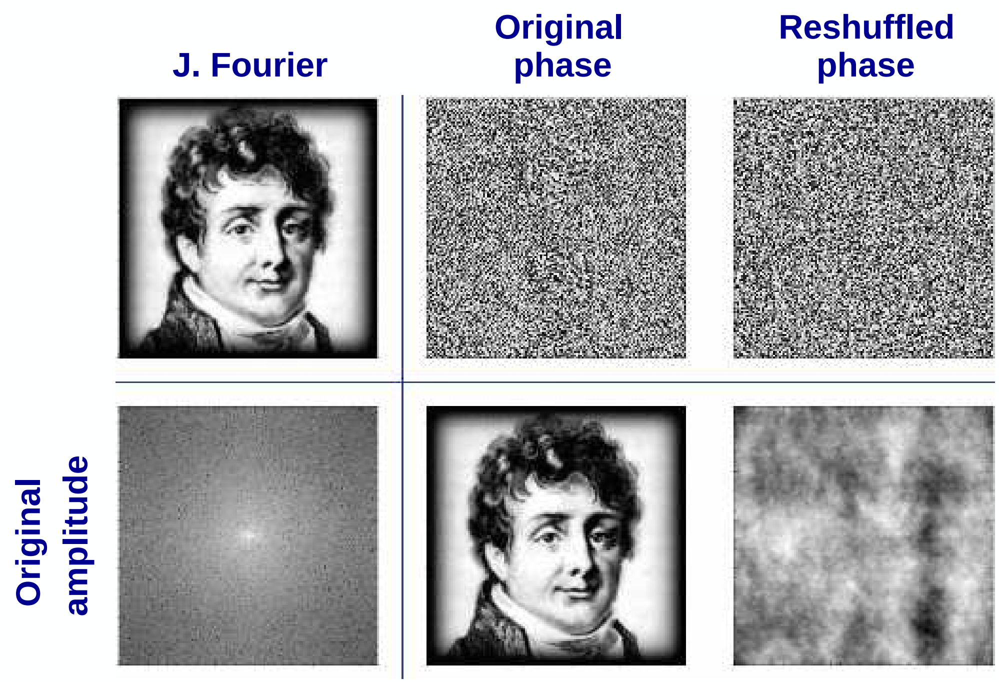
Structure function



Adaptation of statistical tools to the measurement space

- Structure function $\Leftrightarrow$ *Direct space* $\Leftrightarrow$ Single dish *(Bensch et al., 2001)*
- Power spectrum $\Leftrightarrow$ *Fourier space* $\Leftrightarrow$ Aperture synthesis *(Levrier, Ph.D.T., 2004)*
- But power spectra only make use of Fourier amplitudes, not phases...

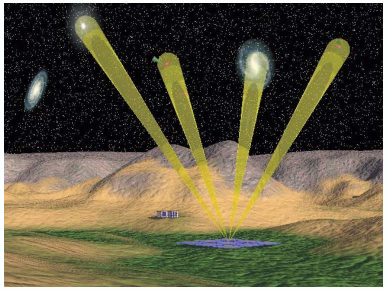
On the importance of Fourier phases



Reshuffling of the Fourier phases

▼

Loss of structural information



Carefully designed phased planar arrays

▼

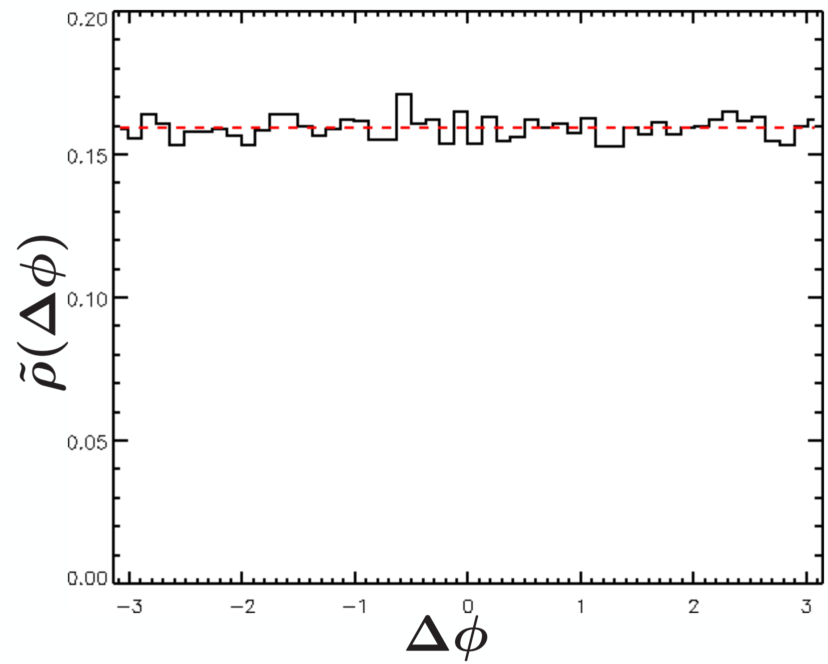
Multibeaming

Phase increments

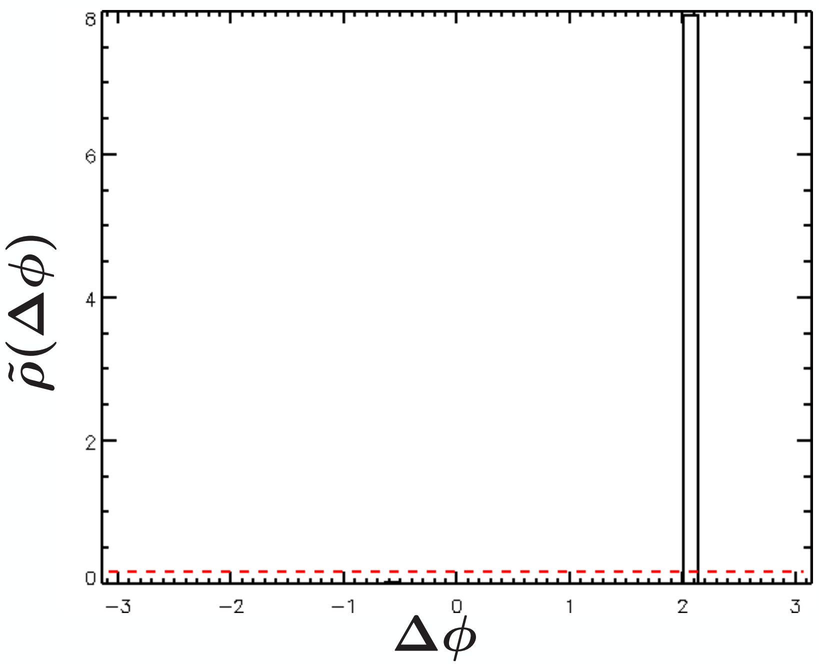
- Defined as $\Delta\phi(\vec{k},\vec{\delta}) = \phi(\vec{k}+\vec{\delta}) - \phi(\vec{k})$ for a given lag vector $\vec{\delta}$
- Statistics of phase increments should trace the structure lost in the reshuffling
- Probability distribution functions $\rho(\Delta\phi)$ approximated by histograms $\tilde{\rho}(\Delta\phi)$

Limiting cases

Fractional Brownian motion



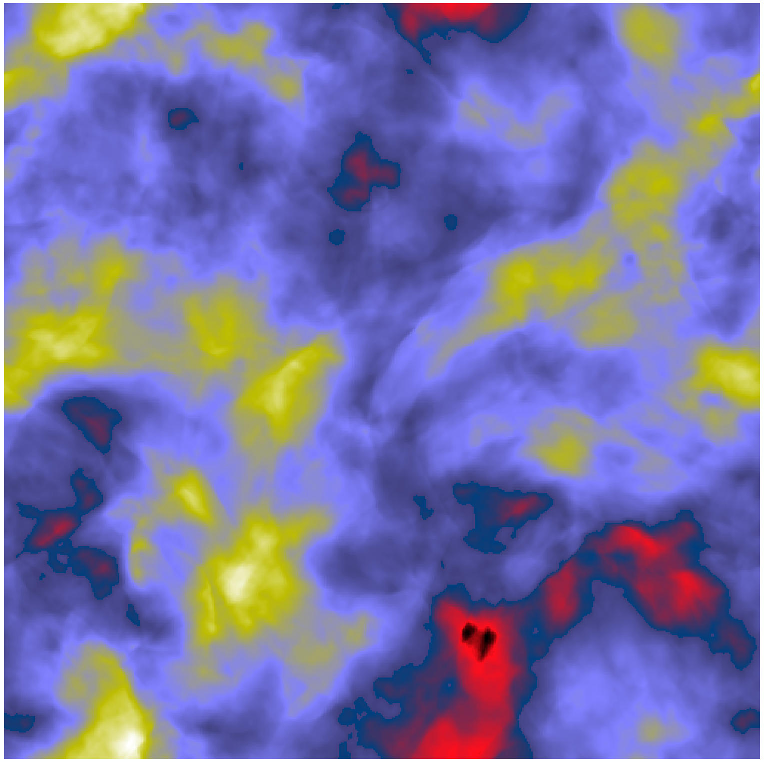
Single point source



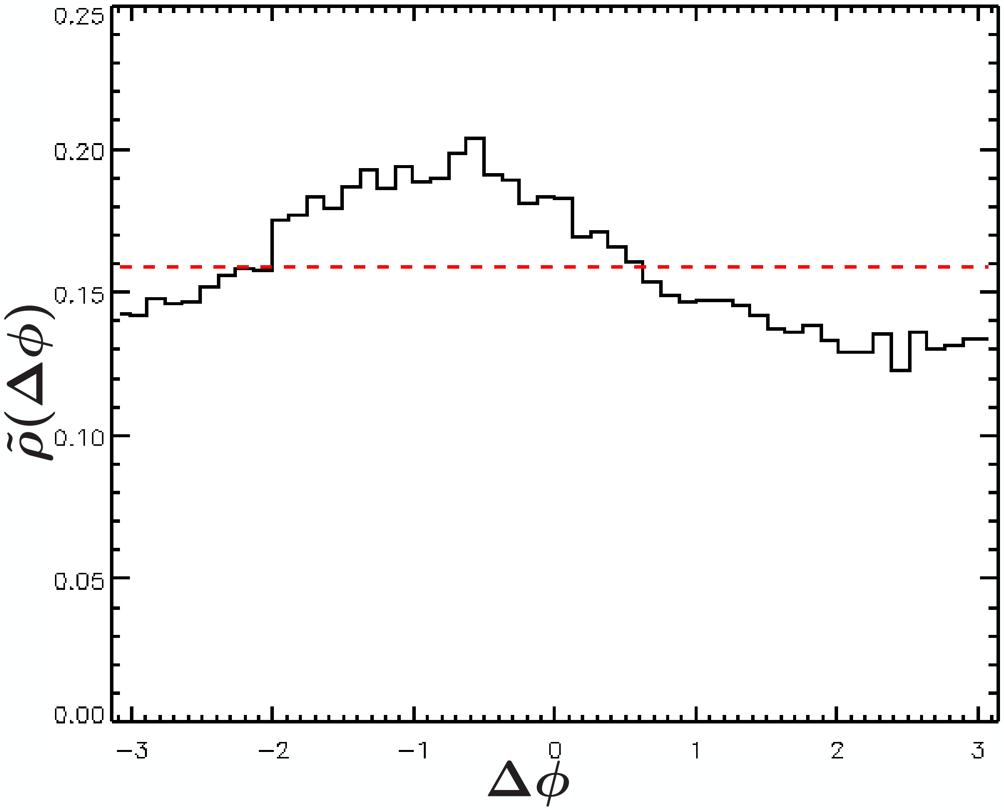
Histograms of phase increments

Compressible hydrodynamical turbulence simulation *(Porter et al., 1994)*

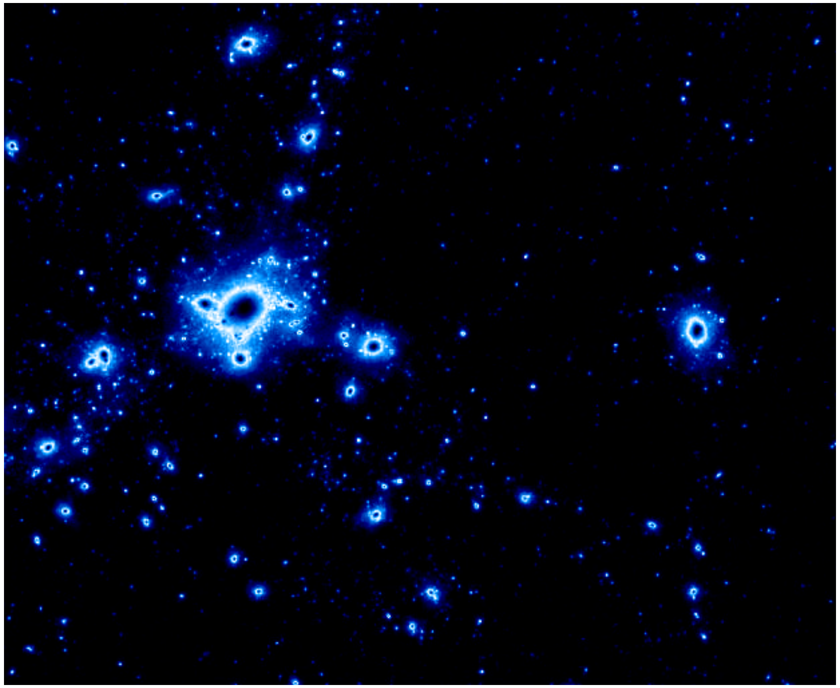
512^2 Column density

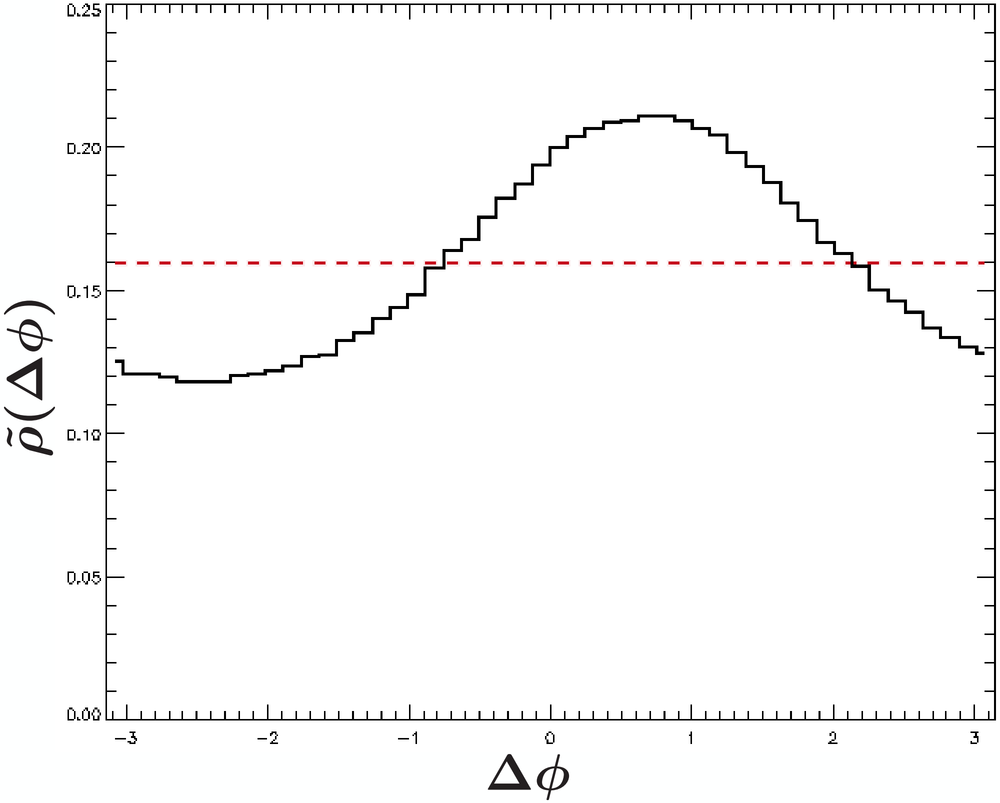


$\vec{\delta} = \vec{e}_x$



Gravitational clustering simulation *(Horizon project)*

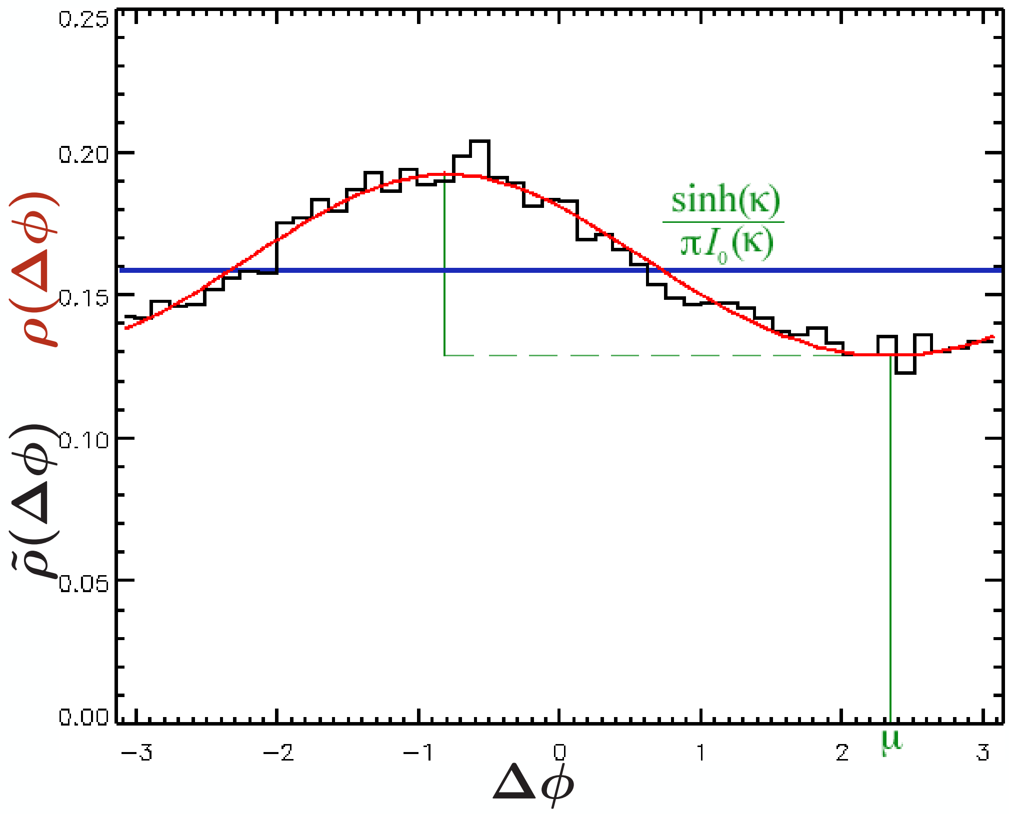




Requires quantification of non-uniformity

$$\rho(\Delta\phi) = \frac{1}{2\pi I_0(\kappa)} e^{-\kappa cos(\Delta\phi - \mu)}$$

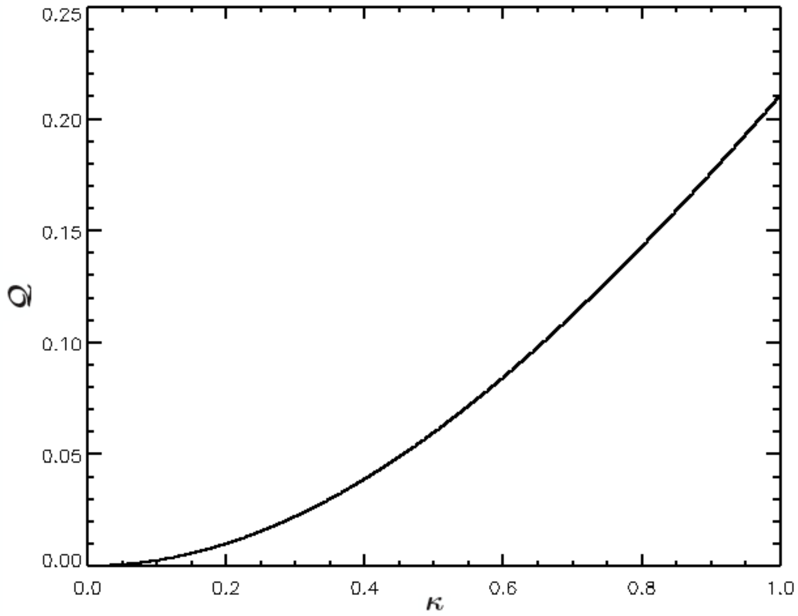
(Watts et al., 2003)



Characterizations of non-uniformity

- **Phase entropy :** $\mathcal{S}(\vec{\delta}) = -\int_{-\pi}^{\pi} \rho(\Delta\phi)\, ln[\rho(\Delta\phi)] d\Delta\phi$ *(Polygiannakis & Moussas, 1995)*
- **"Phase structure quantity" :** $\mathcal{Q}(\vec{\delta}) = ln\,(2\pi) - \mathcal{S}(\vec{\delta}) \geq 0$

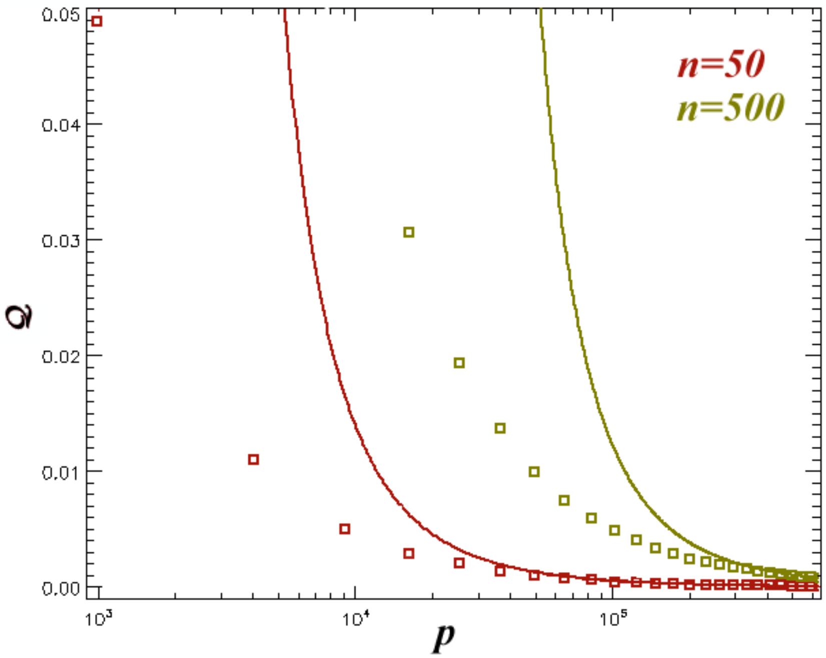
Relation to the von Mises parameter κ



$$\mathcal{Q} = \kappa \frac{I_1(\kappa)}{I_0(\kappa)} - ln\,[I_0(\kappa)]$$

Statistical noise on finite-sized images

- May lead to false detection of phase structure
- Requires an estimate of x so that no phase structure implies $\mathcal{Q} < x$
- Depends on number of phase increments p and number of bins n
- Theoretical upper limit x computed from χ^2 statistics *(Levrier, Falgarone & Viallefond, 2006)*



Primary beam attenuation

- Convolution in Fourier space
- Mosaic observations effectively reduce kernel size

Not considered $\Longleftrightarrow$ Pointlike antennae

Pillbox gridding

- Measured phases associated with "wrong" wavenumber
- Model brightness distributions already gridded

Not considered $\Longleftrightarrow$ Phase constant over each pixel

Atmospheric phase noise

- Atmospheric turbulence makes phase space- and time-dependent

Considered ... See later

Phase structure quantity in observations

The input brightness distribution

Column density of a compressible hydrodynamical simulation for which $\mathcal{Q} \simeq 0.01$

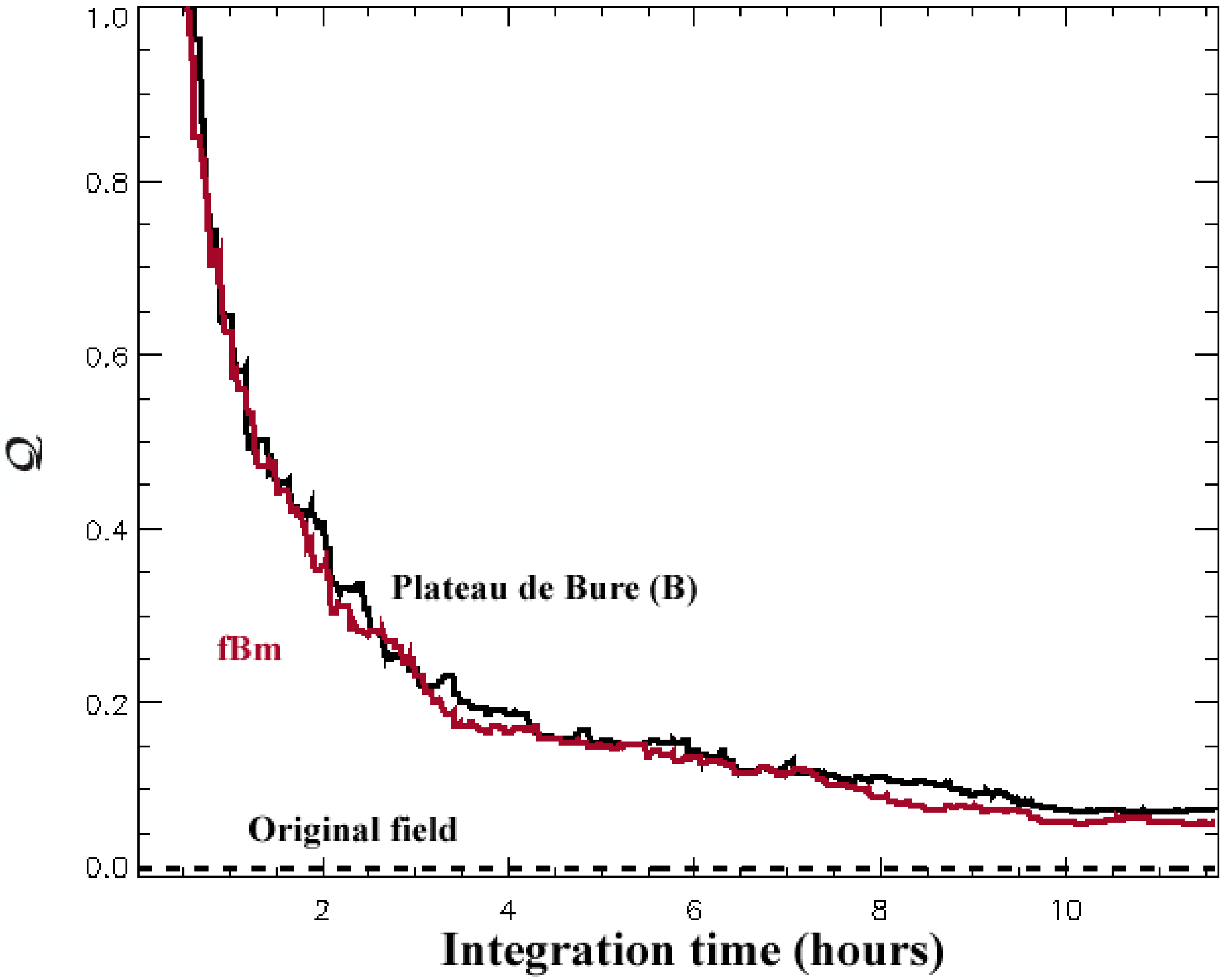
The parameters

- 3 possible arrays
- Single or multiple configurations (possibly trimmed)
- Atmospheric phase noise

The questions asked

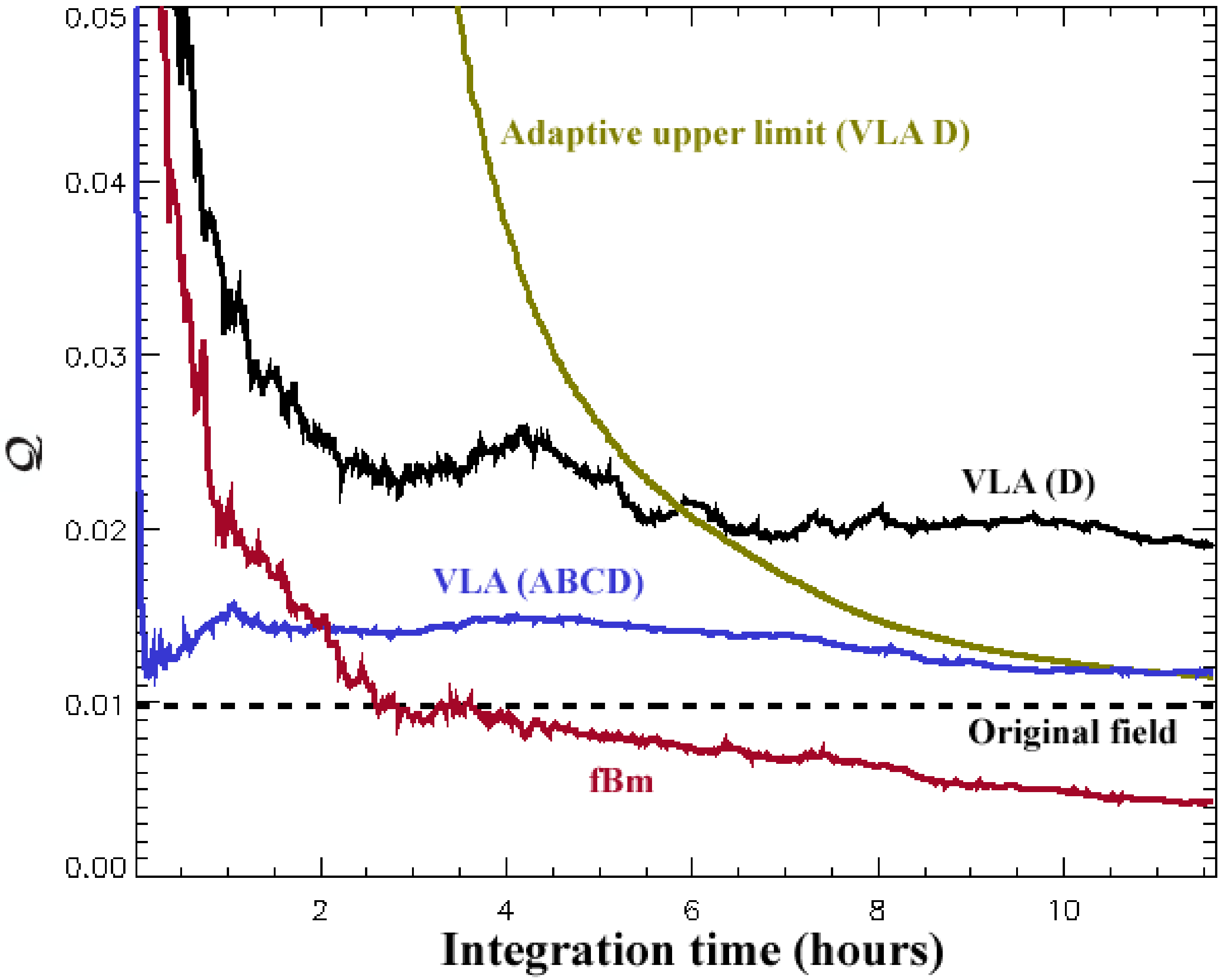
- How long does it take to achieve a significant detection of phase structure ?
- How long does it take to recover the actual phase structure quantity ?
- What level of atmospheric turbulence still allows detection of phase structure ?

Noise-free observations with Plateau de Bure



Detection not possible

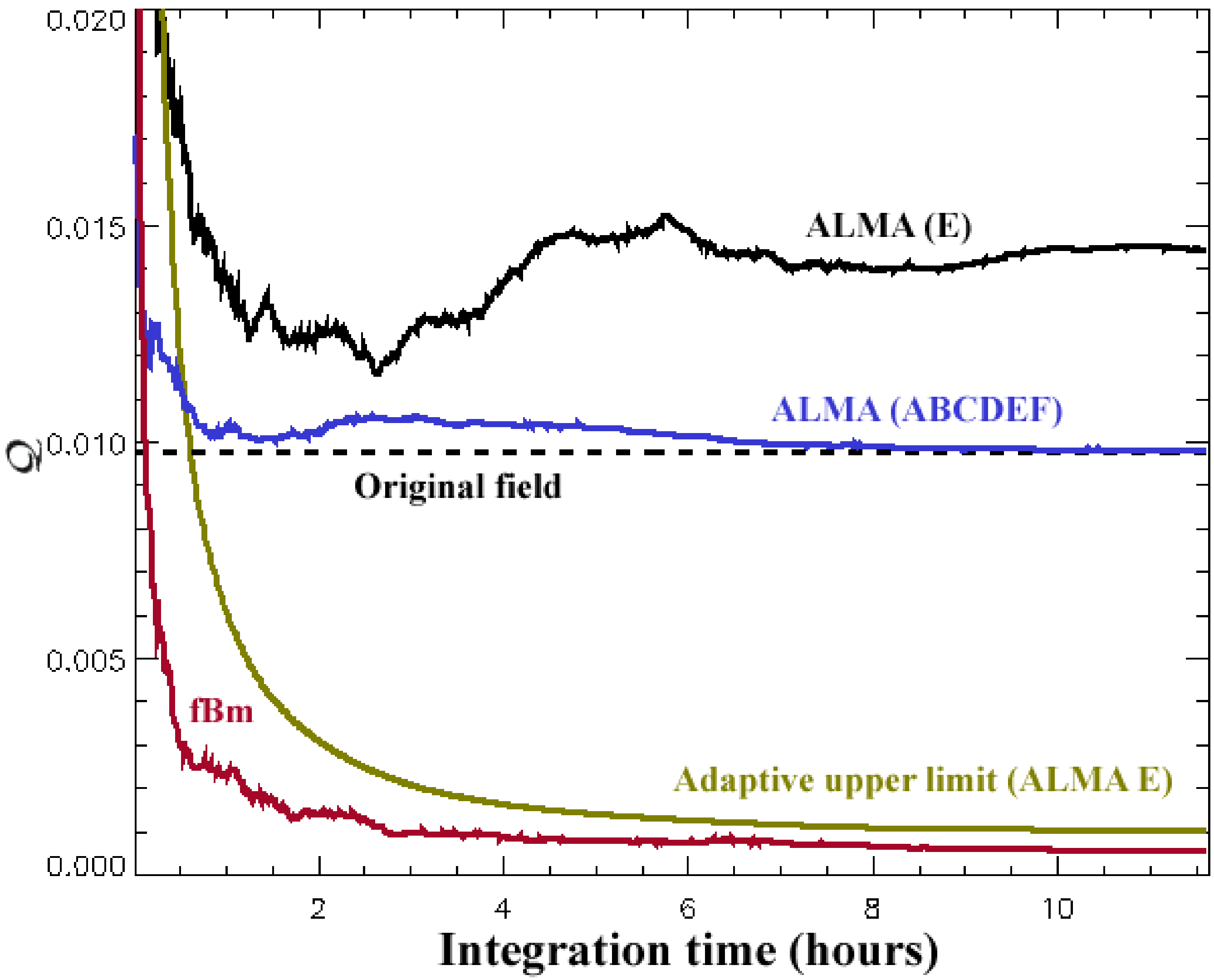
Noise-free observations with the VLA



Detection possible with single configuration

Measurement not possible with multiple configurations

Noise-free observations with ALMA

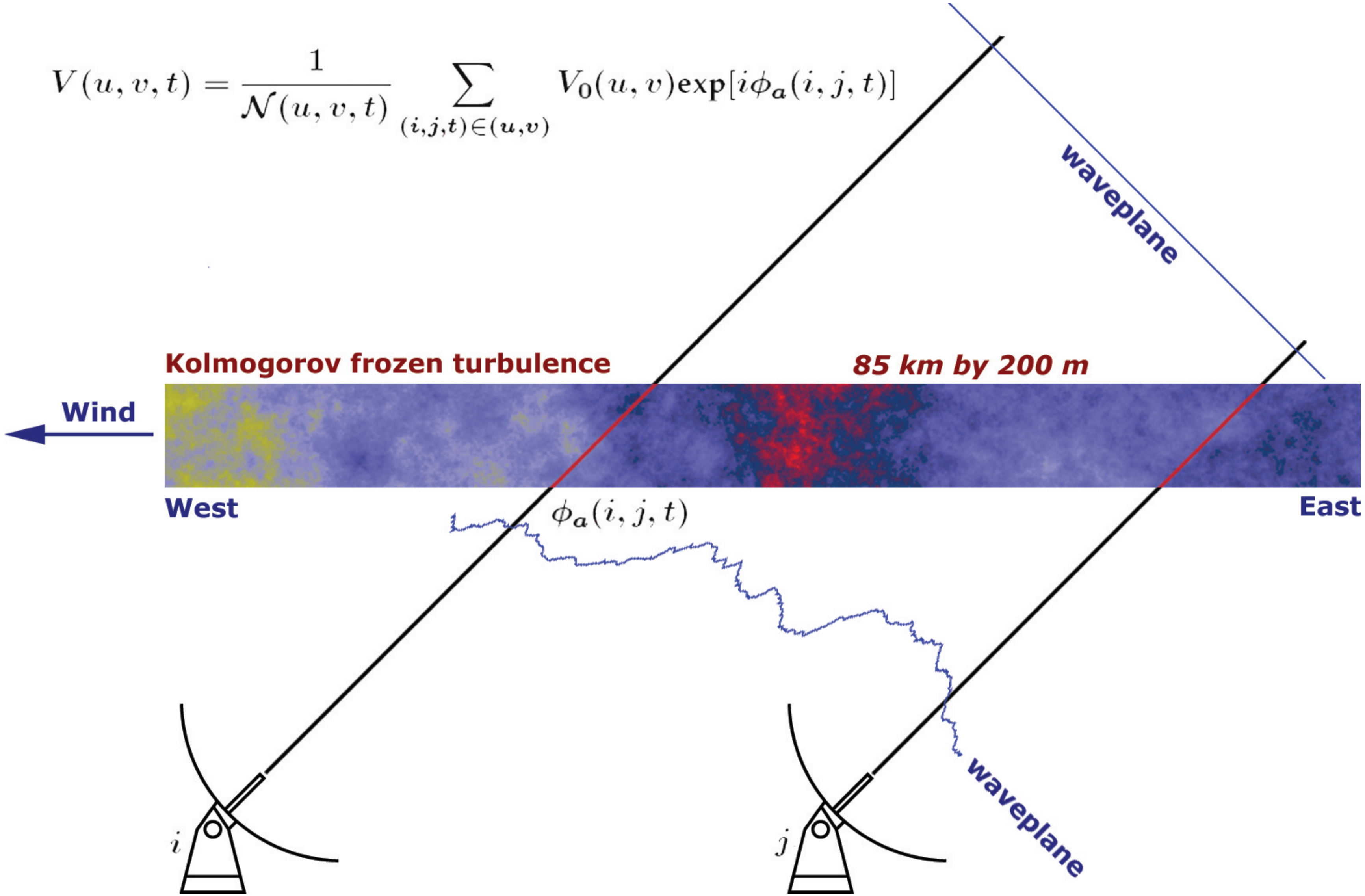


Detection possible with single configuration

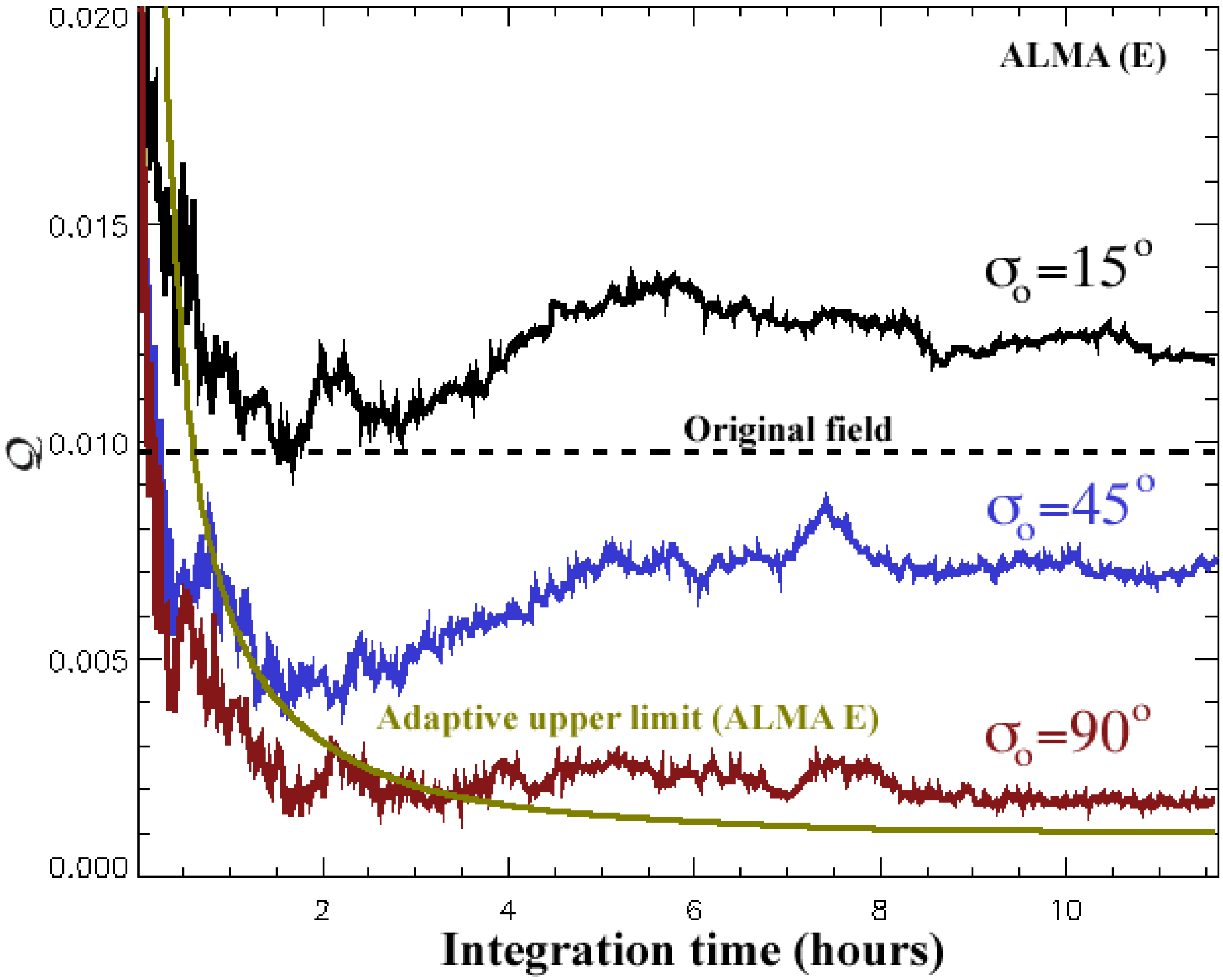
Measurement possible with multiple configurations

Atmospheric phase noise

$$V(u,v,t) = \frac{1}{\mathcal{N}(u,v,t)} \sum_{(i,j,t)\in(u,v)} V_0(u,v)\exp[i\phi_a(i,j,t)]$$



Noisy observations with ALMA



rms phase delay σ_0 :

- 100m baseline
- 1.3 mm wavelength
- Zenith observation

Chajnantor: 15° to 60°

Detection possible with single configuration

Conclusions

Detection of phase structure

- Requires extended ALMA configuration
- Atmospheric phase noise not critical

Measurement of phase structure

- Requires multiple ALMA configurations

Open questions

- Allow for variations of $\vec{\delta}$
- Interpretation of phase structure quantities $\Longleftrightarrow$ Physical processes