Polarized thermal dust emission

Stokes parameters (Lee & Draine 85, Wardle & Königl 90, Fiege & Pudritz 2000, Pelkonen, Juvela & Padoan 2007, 2009)

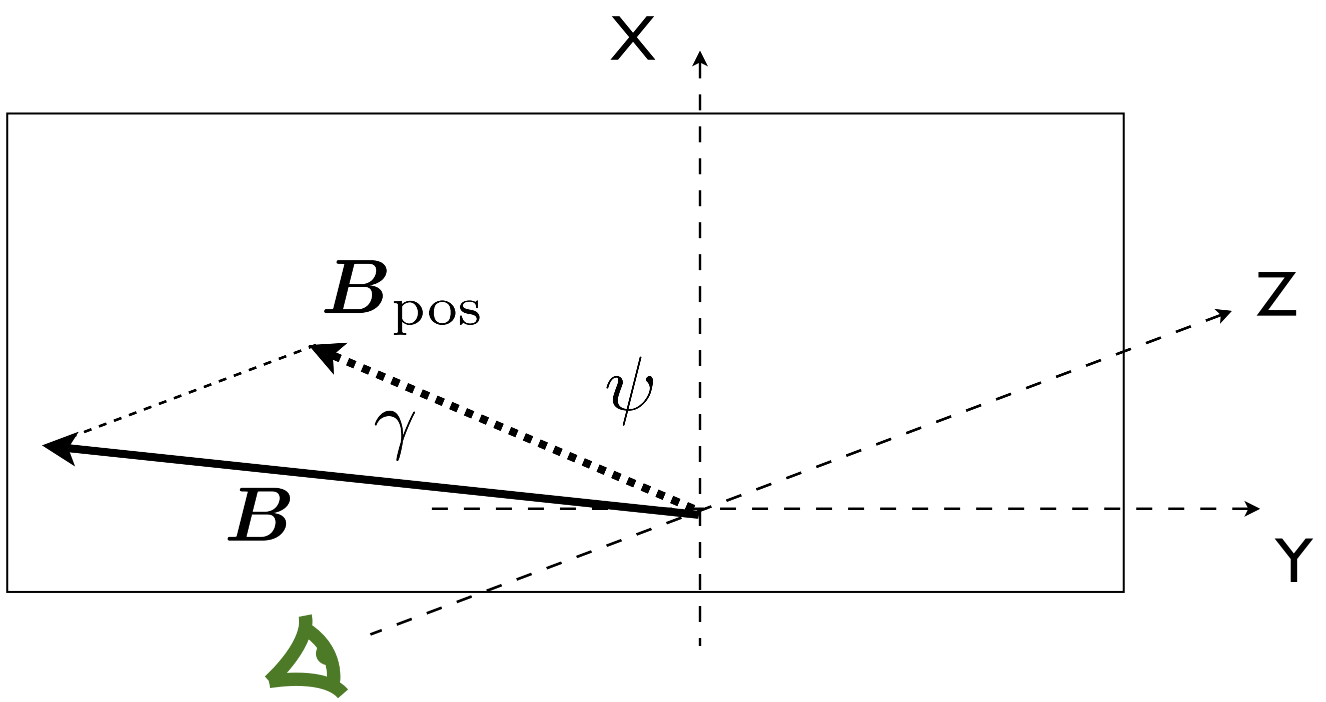
$$I = \int I_\nu \left[1 - \frac{\alpha}{2}\left(\cos^2\gamma - \frac{2}{3}\right)\right] \mathrm{d}z$$

$$Q = \int \alpha I_\nu \cos 2\psi \cos^2\gamma \ \mathrm{d}z$$

$$U = \int \alpha I_\nu \sin 2\psi \cos^2\gamma \ \mathrm{d}z$$

Assumed proportional to density

Taken as intrinsic polarisation (0.1)



Polarised intensity and polarisation angle $P = \sqrt{Q^2 + U^2}$ $\tan 2\chi = \frac{U}{Q}$

How can we retrieve statistics of the turbulent component of B ?

- Build synthetic observables with controlled statistics
- Use numerical MHD simulations to validate inversion methods (n-B correlation)

Synthetic magnetic fields

Synthetic vector potential $\boldsymbol{A}$ $\longrightarrow$ $\boldsymbol{B} = \boldsymbol{\nabla} \times \boldsymbol{A}$

$\longrightarrow$ ***Gaussianity ?***

$\longrightarrow$ ***Power spectra ?***

Components of $\boldsymbol{A}$ are built in Fourier space as isotropic fractional Brownian motions (fBm) :

$\longrightarrow$ ***Power-law amplitudes*** $\quad \mathcal{F}_\lambda(\boldsymbol{k}) = \mathcal{F}_0 |\boldsymbol{k}|^{-\beta/2}$

$\longrightarrow$ ***Odd random phases*** $\quad \Phi_\lambda(-\boldsymbol{k}) = -\Phi_\lambda(\boldsymbol{k})$

$$\left.\begin{array}{r} \mathcal{F}_\lambda(\boldsymbol{k}) = \mathcal{F}_0 |\boldsymbol{k}|^{-\beta/2} \\ \Phi_\lambda(-\boldsymbol{k}) = -\Phi_\lambda(\boldsymbol{k}) \end{array}\right\} \longrightarrow \widetilde{A_\lambda}(\boldsymbol{k}) = \mathcal{F}_\lambda(\boldsymbol{k}) \exp\left[i\Phi_\lambda(\boldsymbol{k})\right]$$

Then :

$$\left.\begin{array}{l} B_\lambda = \epsilon_{\lambda\mu\nu}\partial_\mu A_\nu \\ \widetilde{\partial_\lambda F} = ik_\lambda \widetilde{F} \end{array}\right\} \longrightarrow \widetilde{B_\lambda} = \epsilon_{\lambda\mu\nu}\widetilde{\partial_\mu A_\nu} = \epsilon_{\lambda\mu\nu} ik_\mu \widetilde{A_\nu} = \epsilon_{\lambda\mu\nu} ik_\mu \mathcal{F}_\nu(\boldsymbol{k}) \exp\left[i\Phi_\nu(\boldsymbol{k})\right]$$

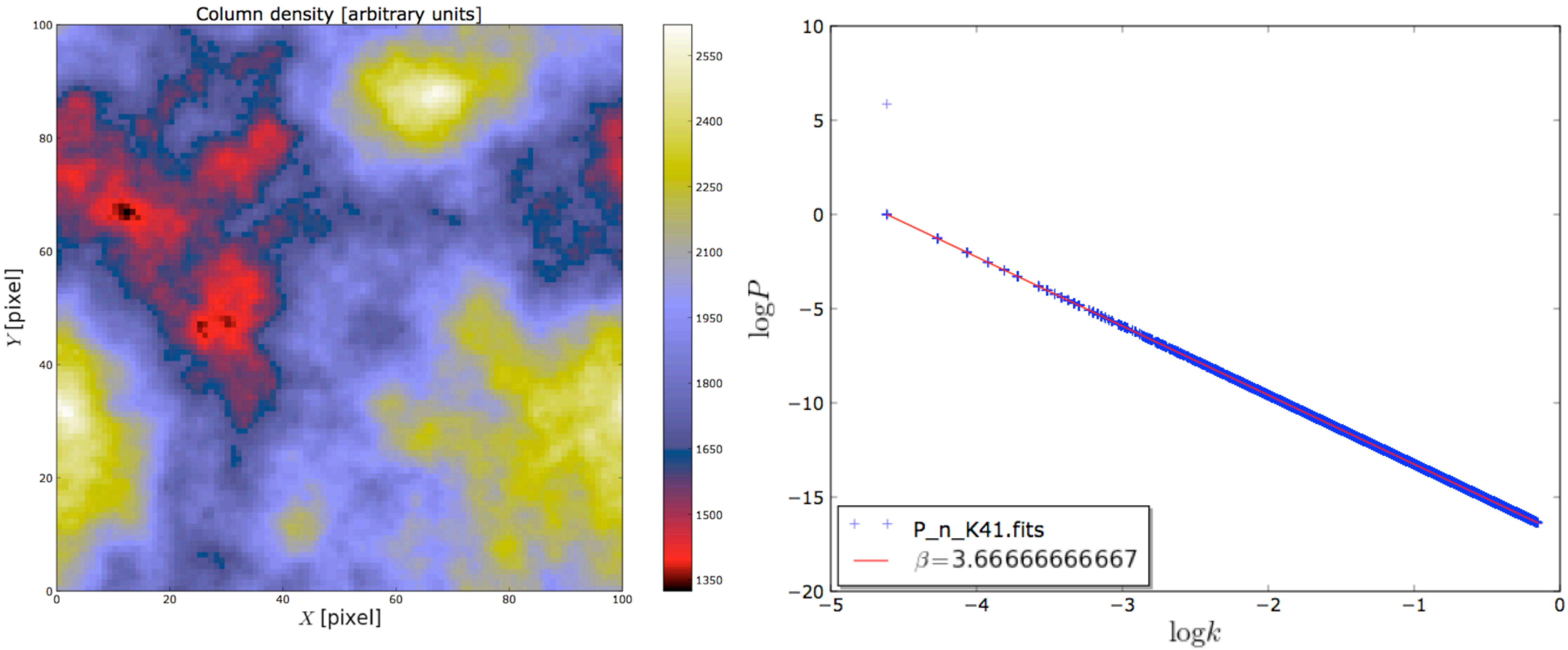
And magnetic field divergence is null, as it should be

$$\partial_\lambda B_\lambda = 0 \Longleftrightarrow ik_\lambda \widetilde{B_\lambda} = 0 \Longleftrightarrow \epsilon_{\lambda\mu\nu} k_\lambda k_\mu \mathcal{F}_0 |\boldsymbol{k}|^{-\beta/2} \exp\left[i\Phi_\nu(\boldsymbol{k})\right] = 0$$

No large-scale field - No scale separation

$$\langle B_i \rangle \simeq 0 \qquad\qquad \sigma_{B_i} \simeq 1$$

Synthetic density field



- Isotropic fractional Brownian motion with power-law power spectrum
- Low fluctuation level
- No correlation with the magnetic field

$$\frac{\sigma_n}{\langle n \rangle} \simeq 0.25$$

B components power spectra

Theoretically :

$$P_{B_\lambda}(\boldsymbol{k}) = |\widetilde{B_\lambda}|^2 = |\epsilon_{\lambda\mu\nu} i k_\mu \mathcal{F}_\nu(\boldsymbol{k}) \exp\left[i\Phi_\nu(\boldsymbol{k})\right]|^2 = \mathcal{F}_0^2 |\boldsymbol{k}|^{-\beta} |\epsilon_{\lambda\mu\nu} i k_\mu \exp\left[i\Phi_\nu(\boldsymbol{k})\right]|^2$$

One can write : $k_\mu = |\boldsymbol{k}| f_\mu \longrightarrow P_{B_\lambda}(\boldsymbol{k}) = \mathcal{F}_0^2 |\boldsymbol{k}|^{2-\beta} |\epsilon_{\lambda\mu\nu} i f_\mu \exp\left[i\Phi_\nu(\boldsymbol{k})\right]|^2$

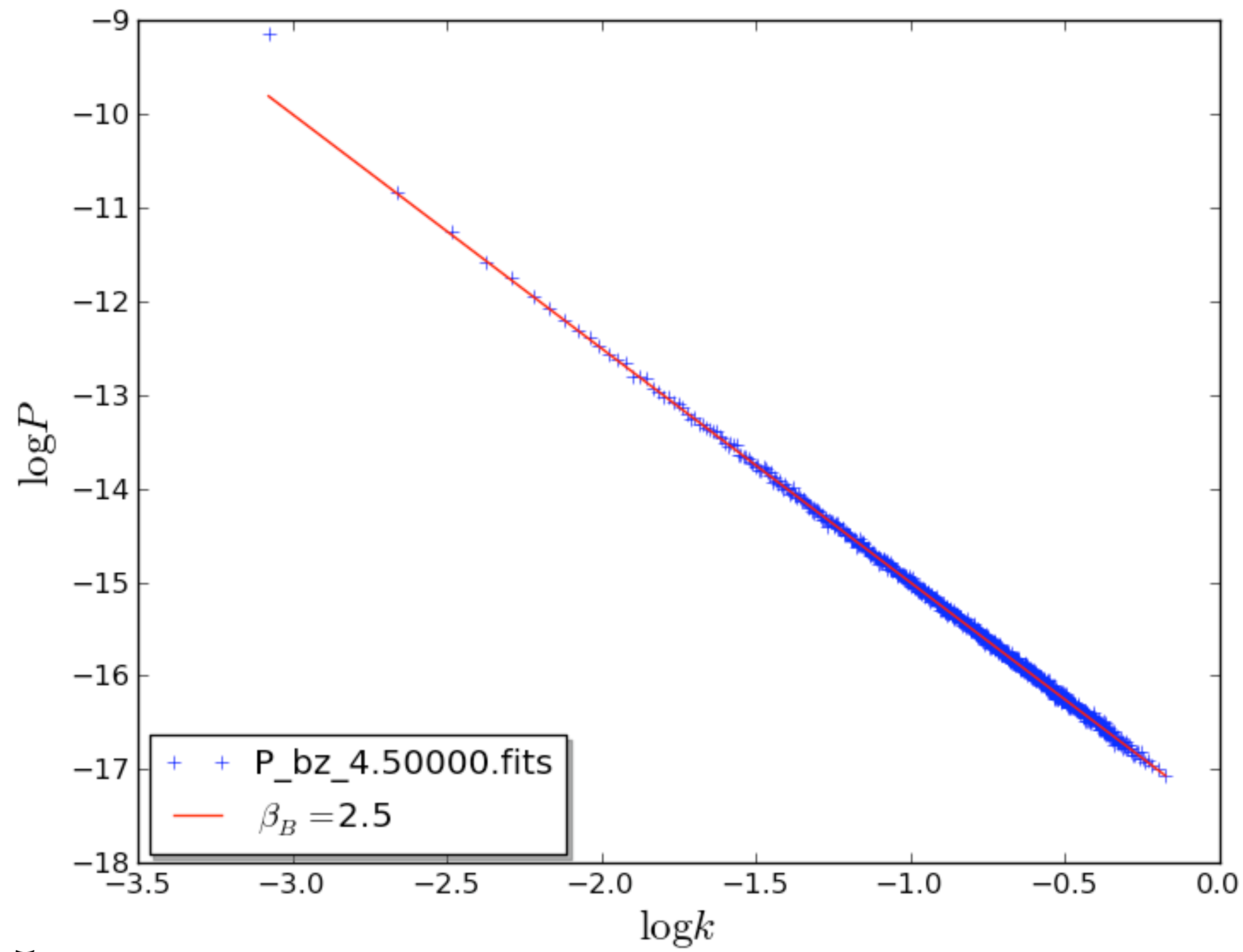
For instance in spherical coordinates : $f_x = \sin\theta\cos\phi$ $\quad f_y = \sin\theta\sin\phi$ $\quad f_z = \cos\theta$

So when averaging at constant wavenumber :

$$\boxed{\langle P_{B_\lambda}(\boldsymbol{k})\rangle_{|\boldsymbol{k}|=k_0} \propto k_0^{2-\beta}}$$

Numerics check out...

β	β_B	
3.1	1.1	✓
3.5	1.5	✓
4.0	2.0	✓
4.5	2.5	✓
4.9	2.9	✓



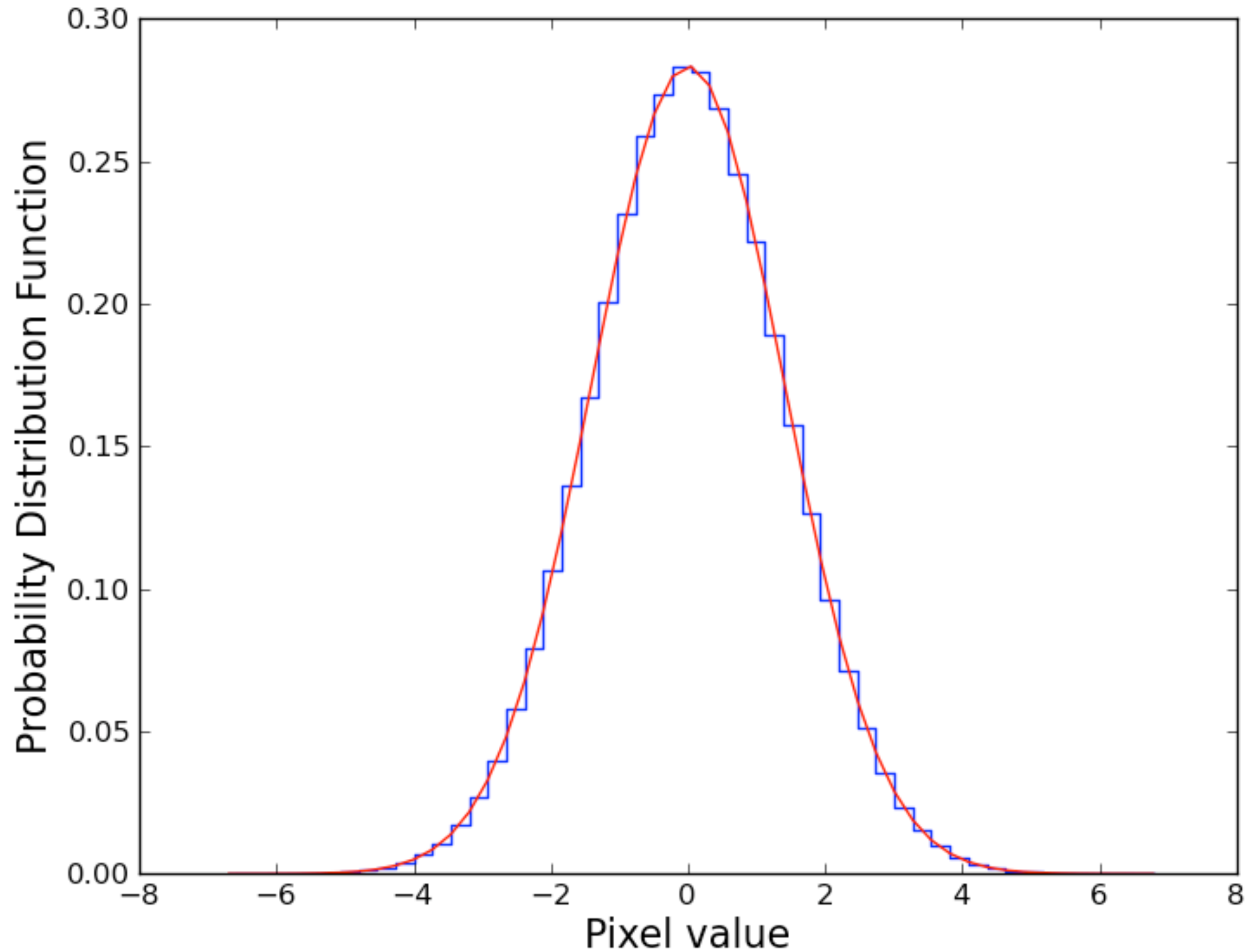
Also valid outside of fBm range $3 \leqslant \beta \leqslant 5$

Gaussianity of B components

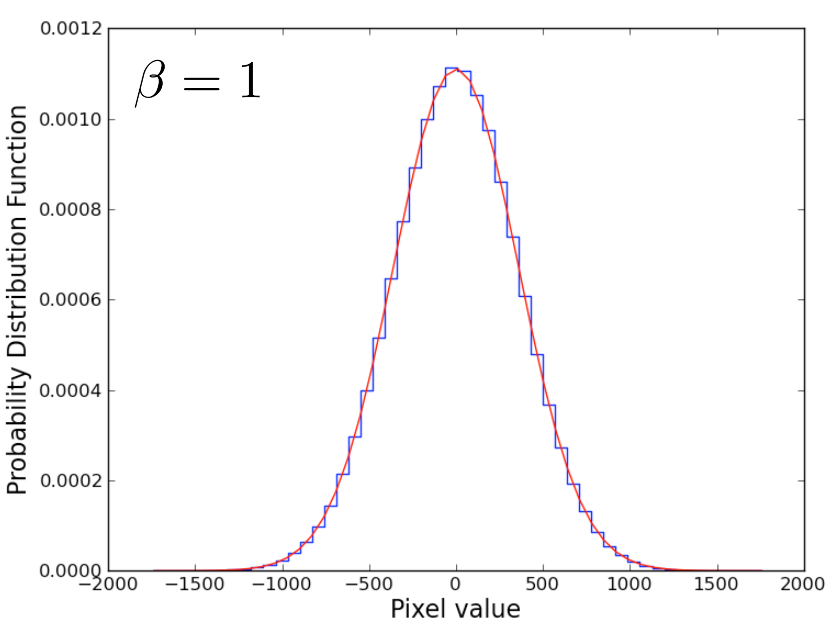
Gaussian in the fBm range

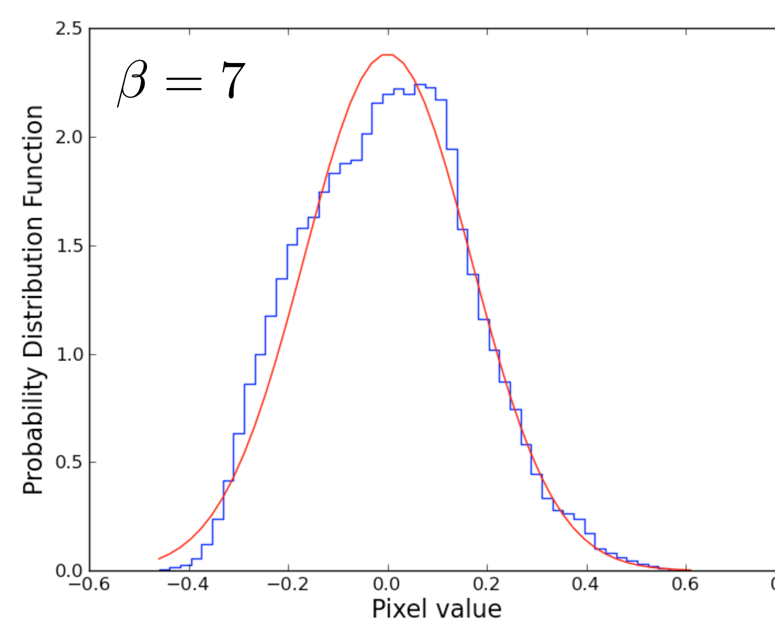
$$3 \leqslant \beta \leqslant 5$$

β	
3.1	✓
3.5	✓
4.0	✓
4.5	✓
4.9	✓

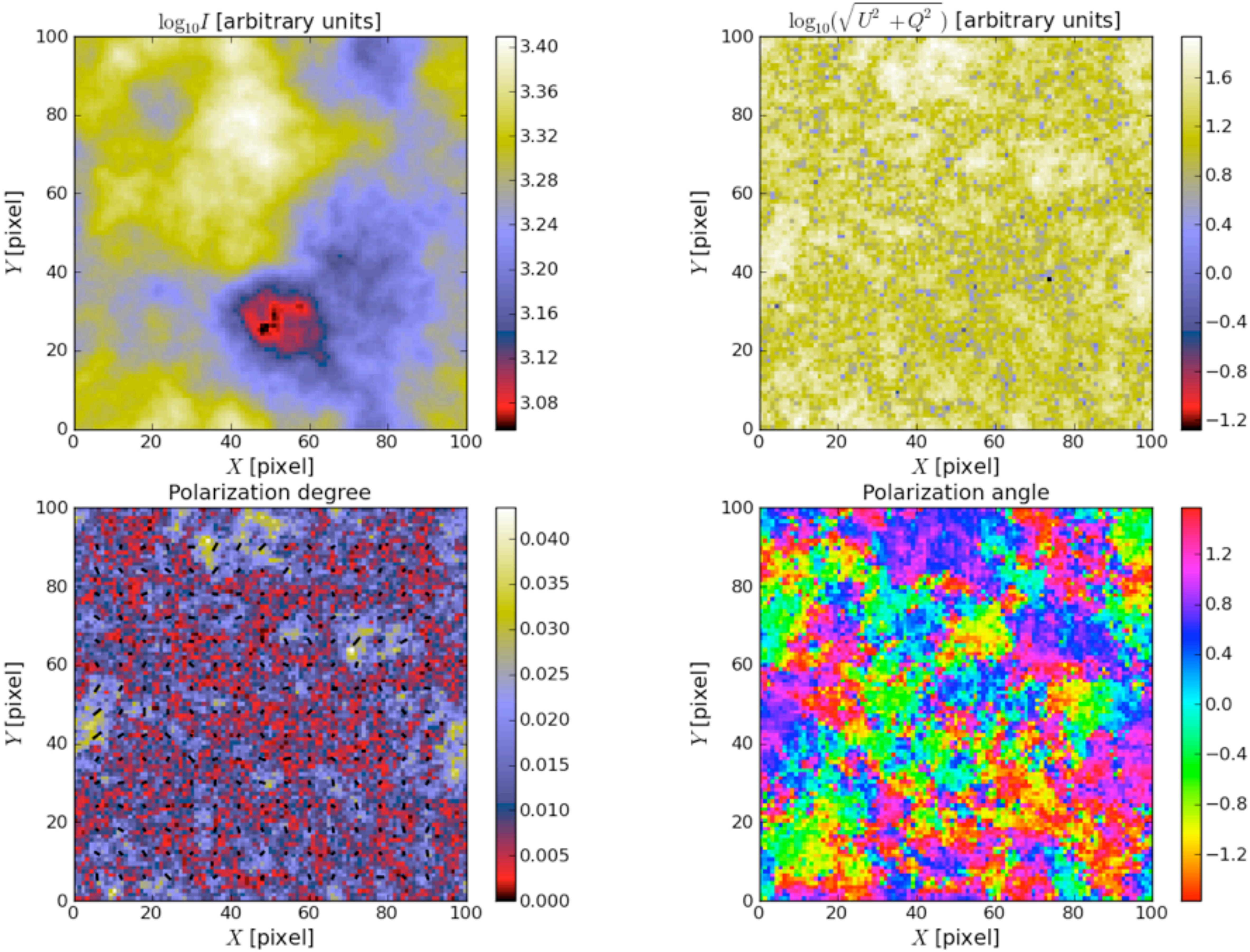


Outside the fBm range, gaussianity is preserved for $\beta \leqslant 3$ but not for $\beta \geqslant 5$

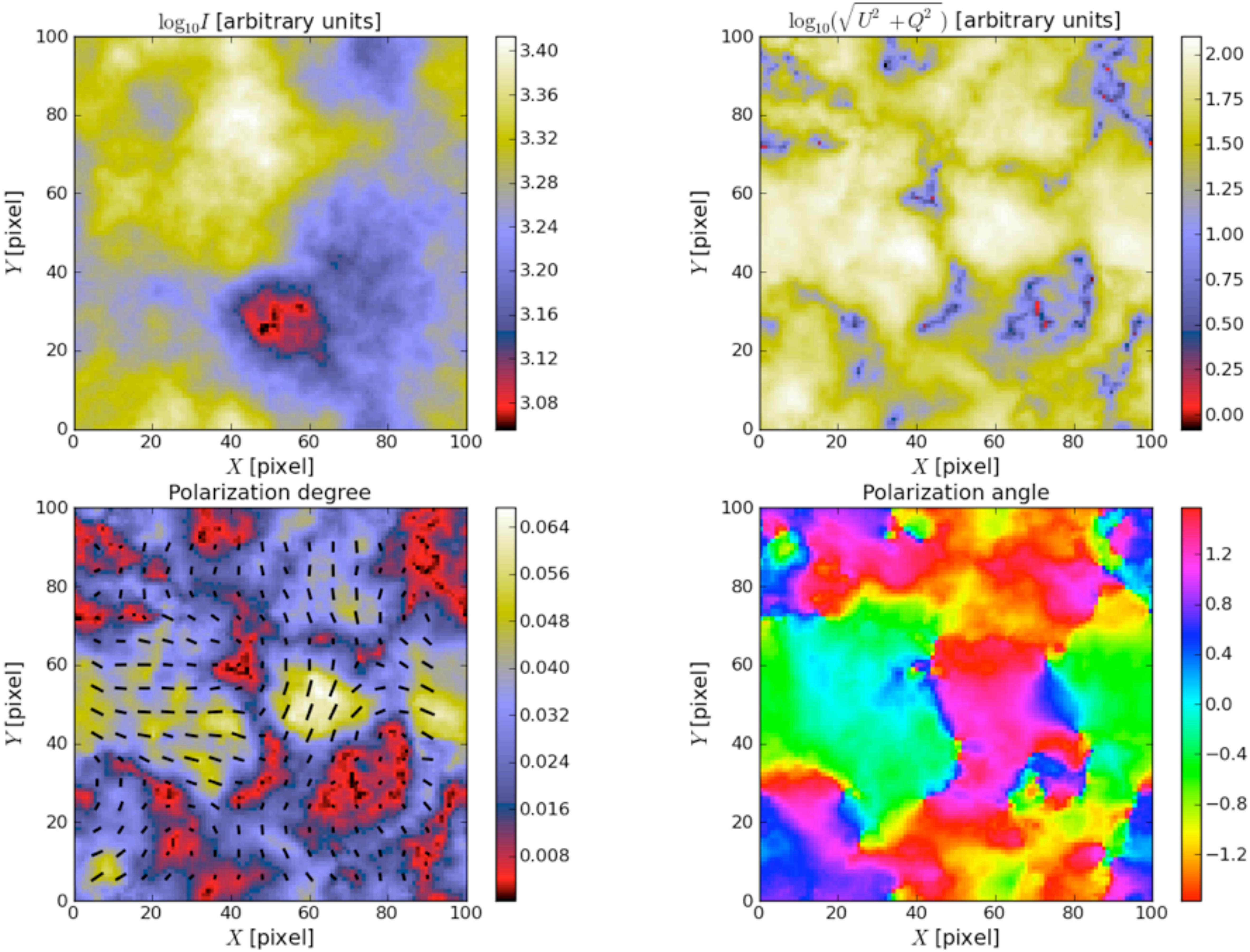




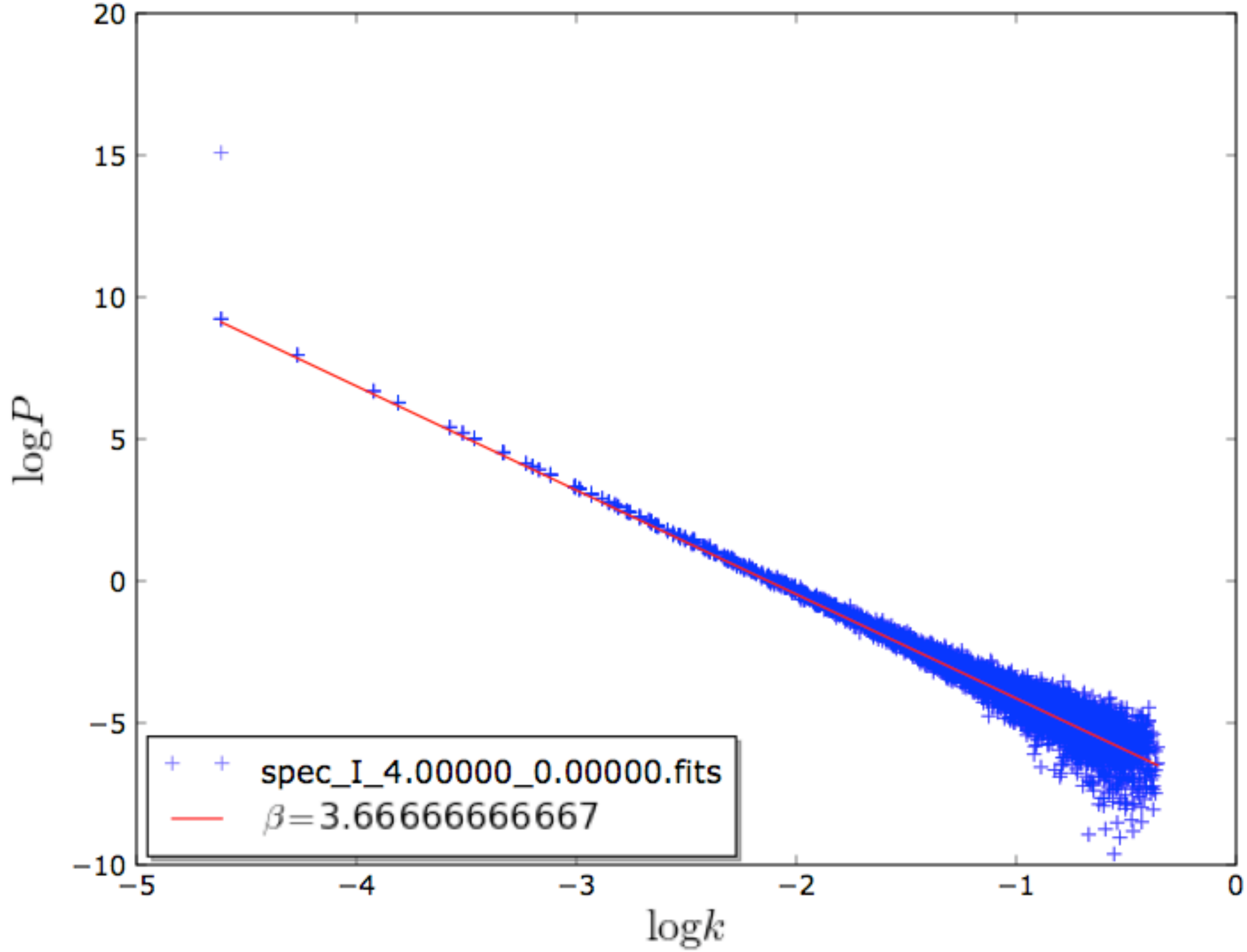
Simulated polarized emission $\beta_B = 2.9$



Simulated polarized emission $\beta_B = 4.5$

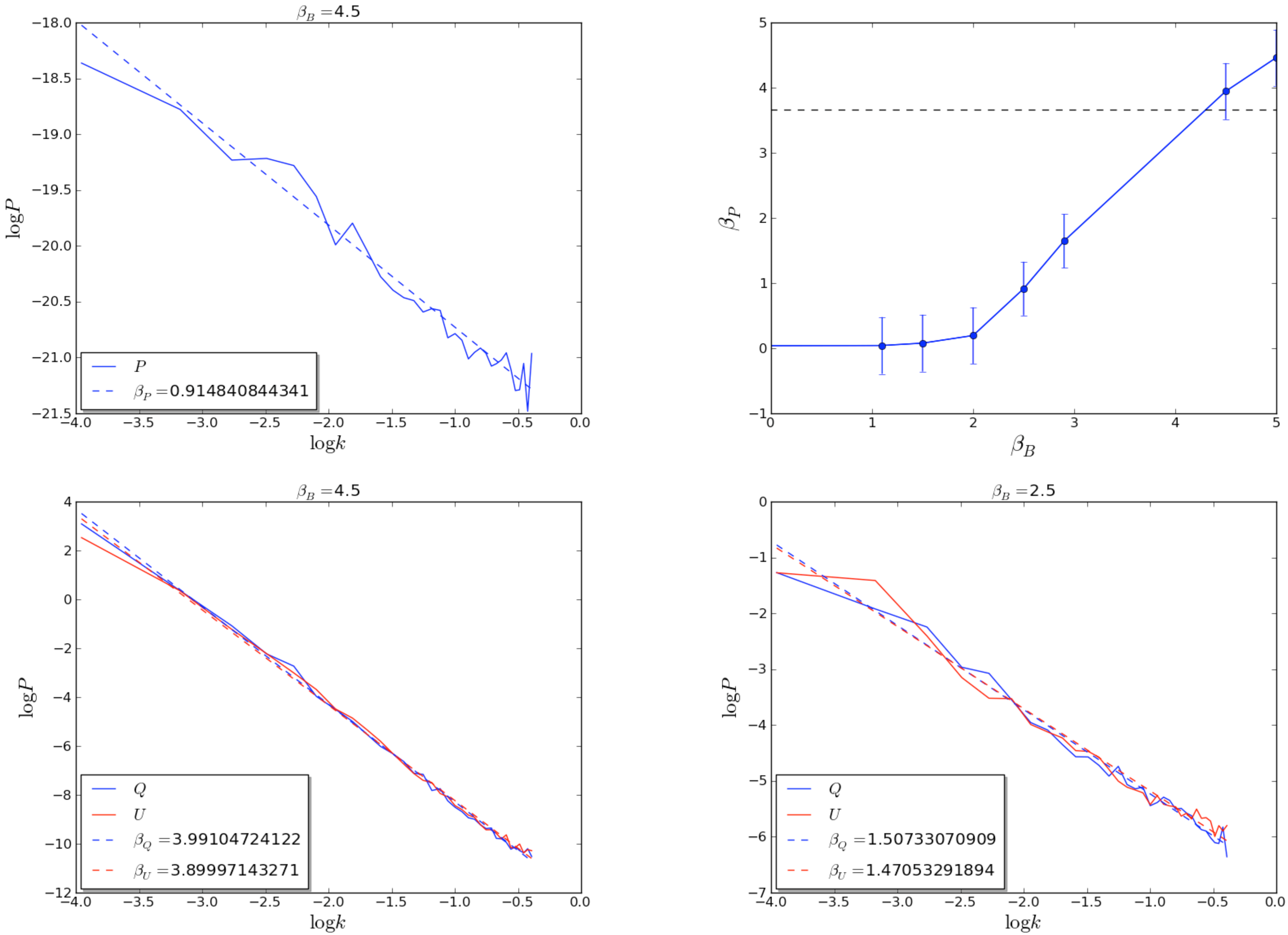


Power spectrum of I



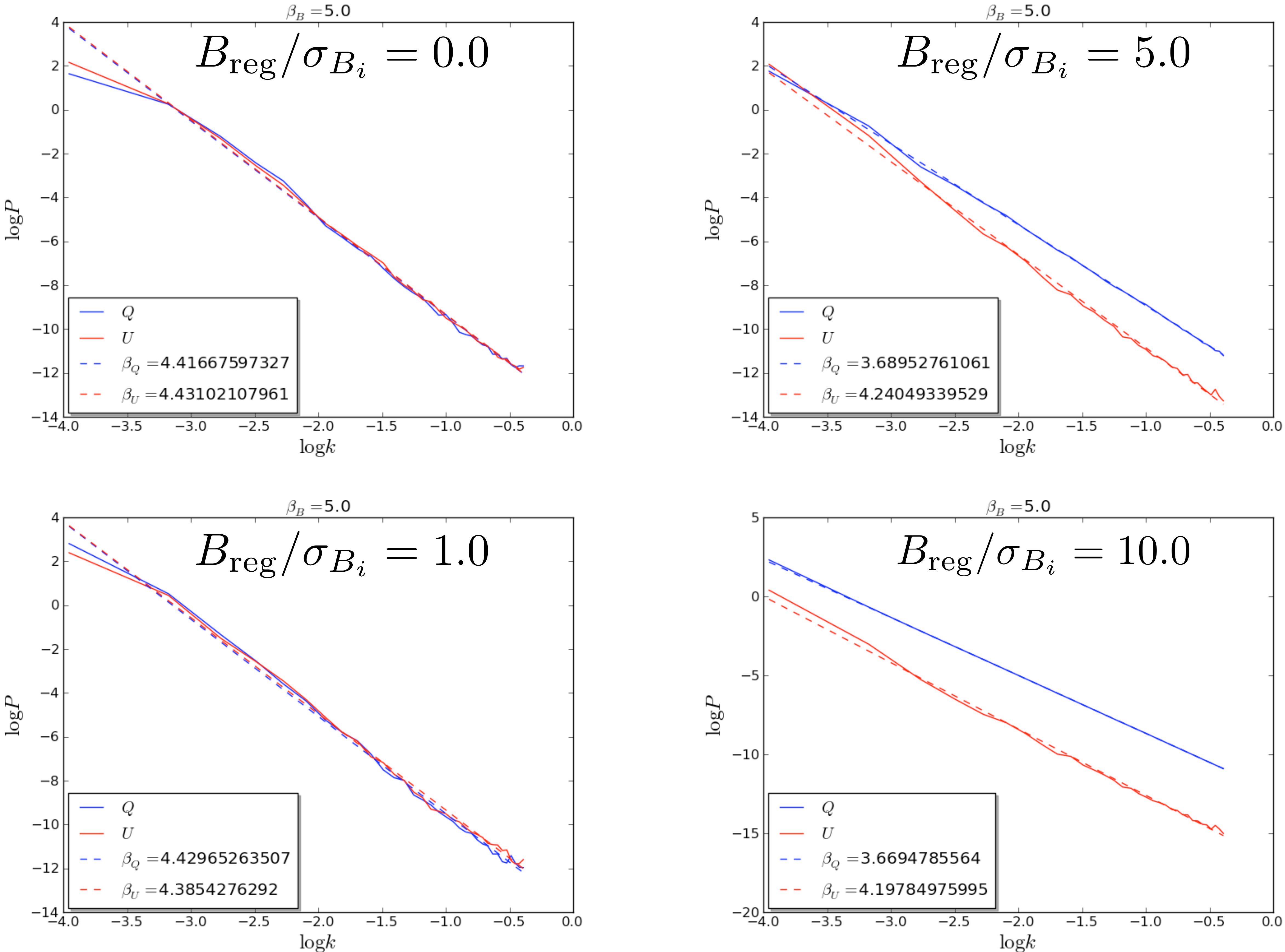
Density field index recovered for all B spectra indices

Power spectra of P, Q and U

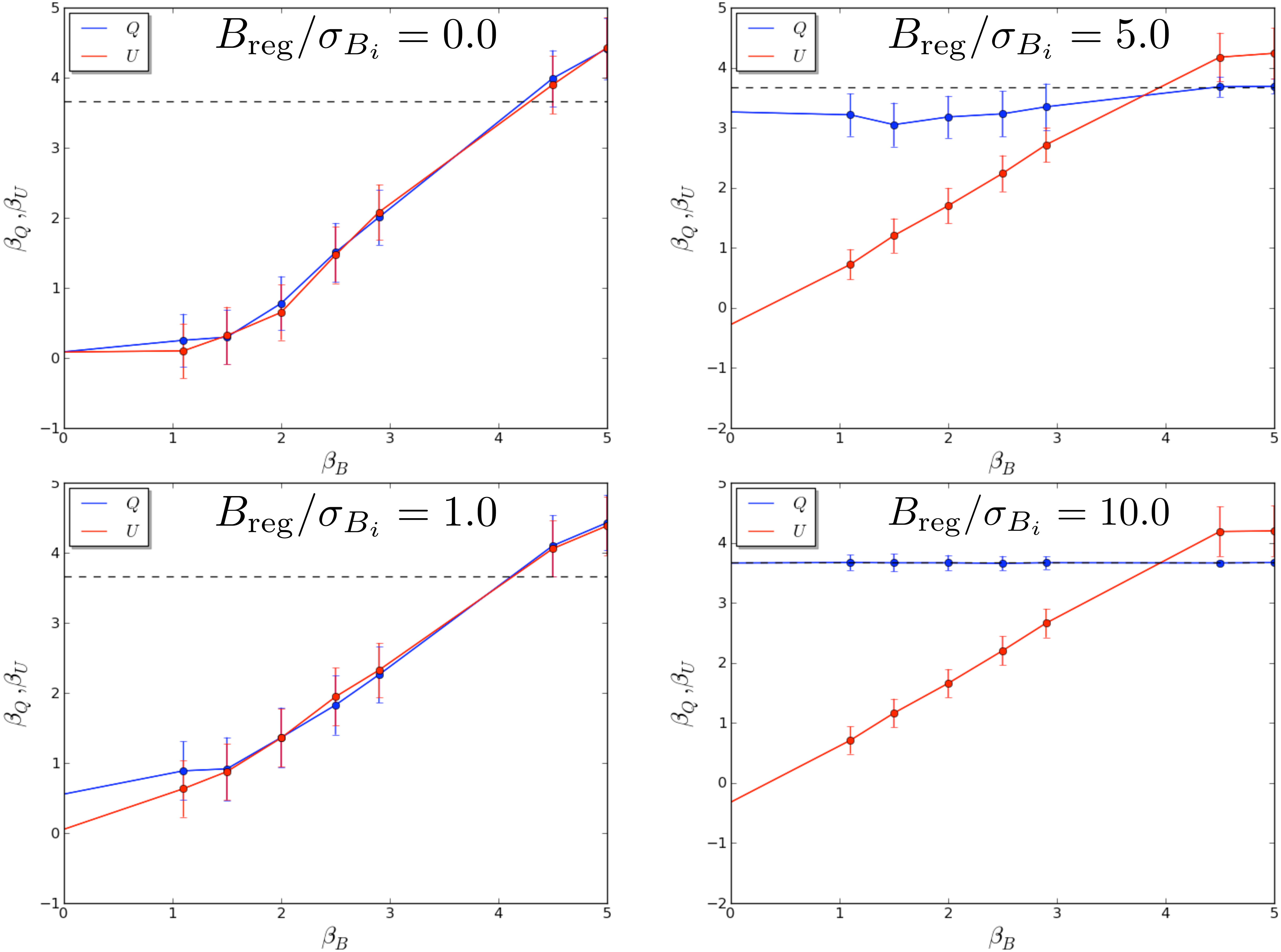


Influence of large-scale field

Inclusion of a uniform magnetic field along X axis (within POS)

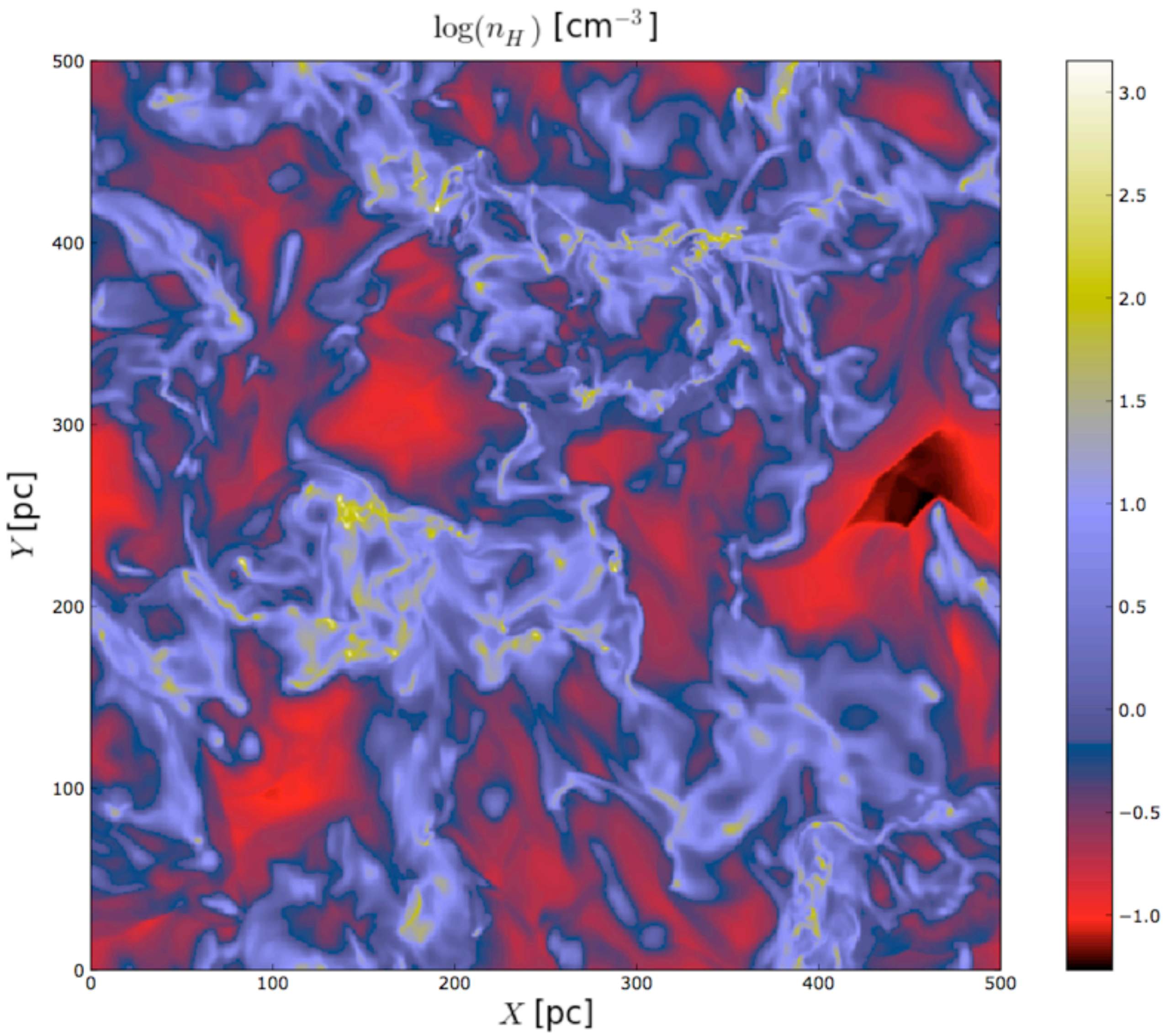


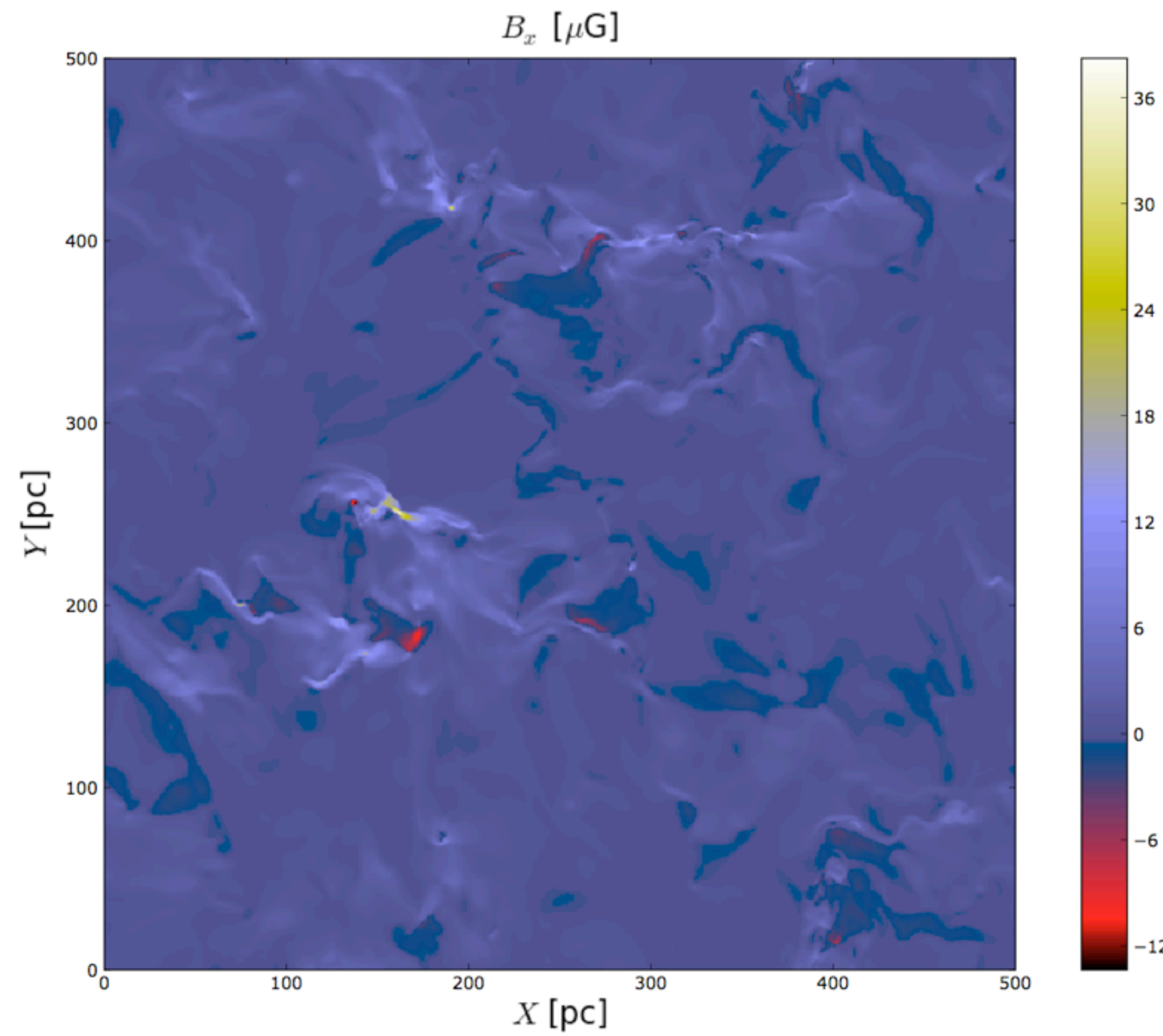
Influence of large-scale field



MHD simulation

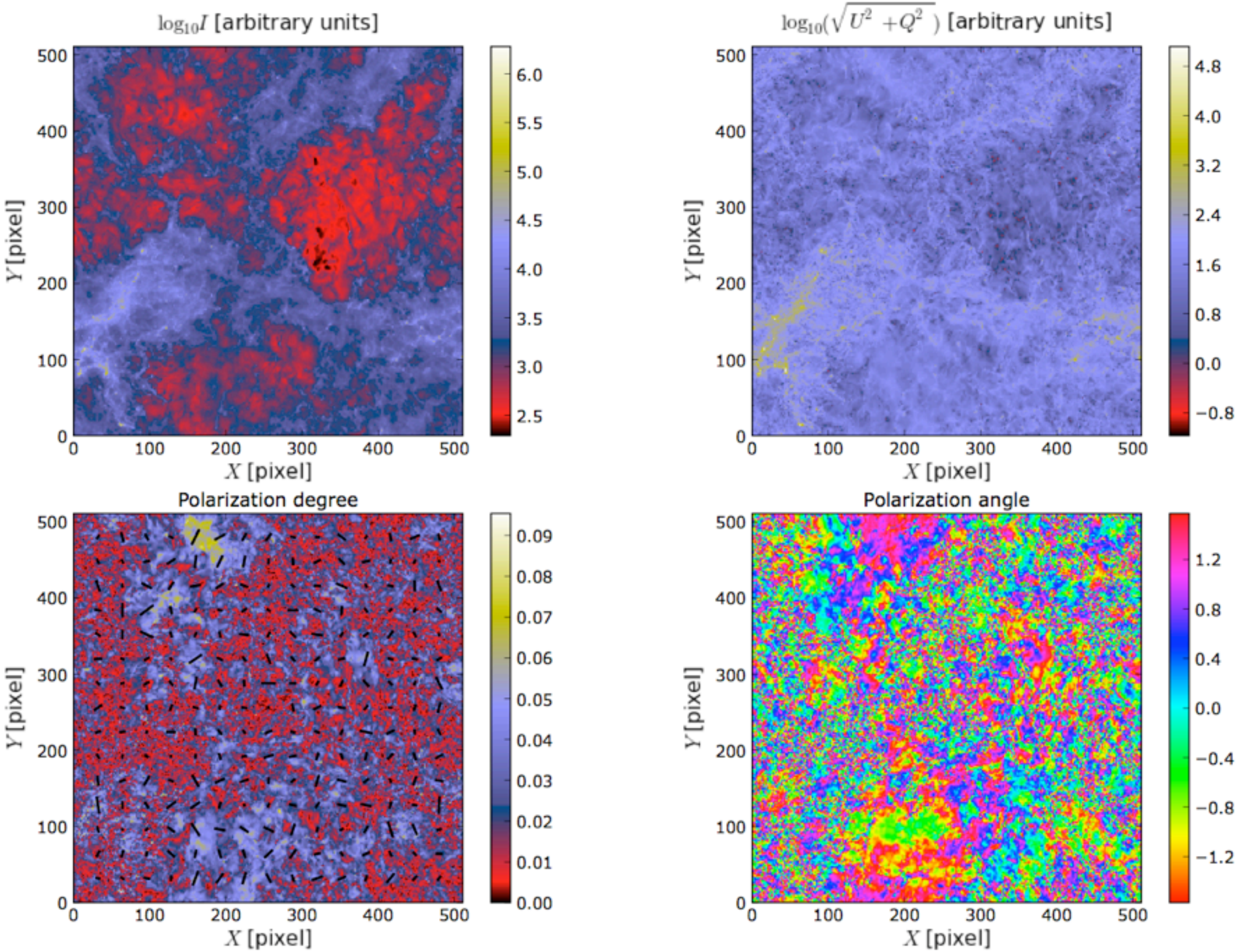
Courtesy of P. Hennebelle



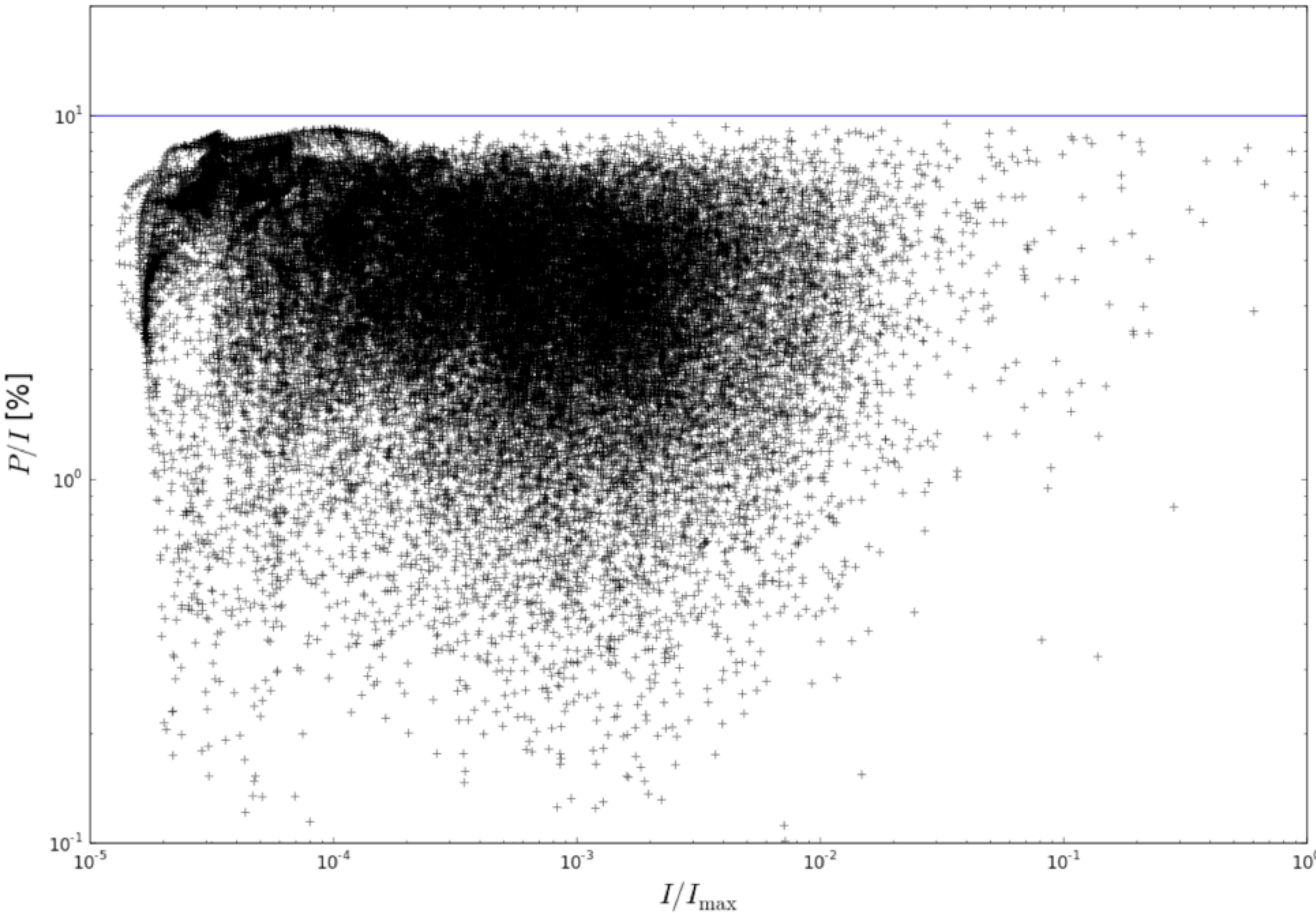


512³ simulation of decaying turbulence
Box size : 500 pc
Mean density : 5 cm-3
Initial rms velocity : 20 km/s
Initial B : 2μG

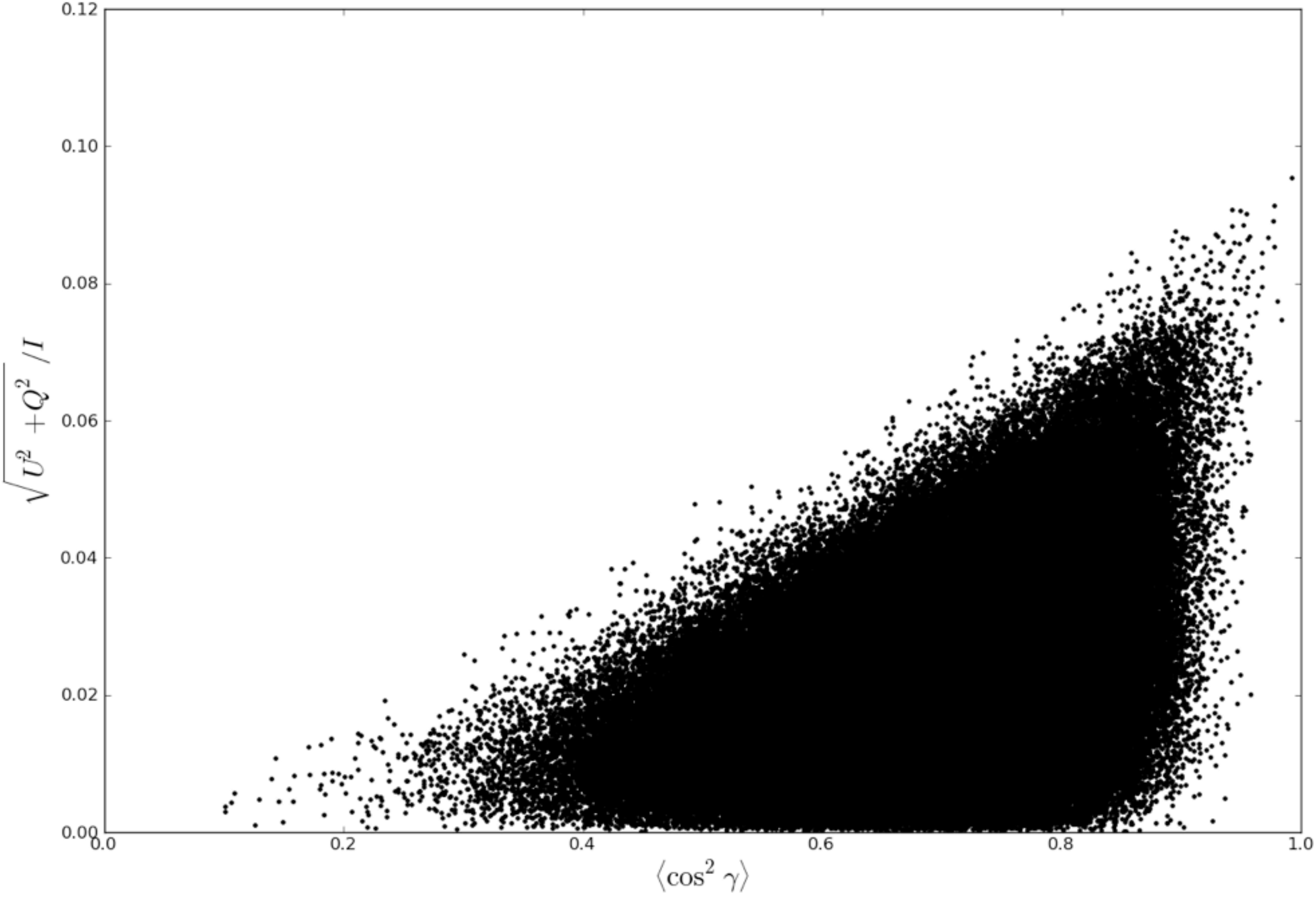
Simulated polarized emission



P-I correlation



Polarisation degree vs B angle on LOS

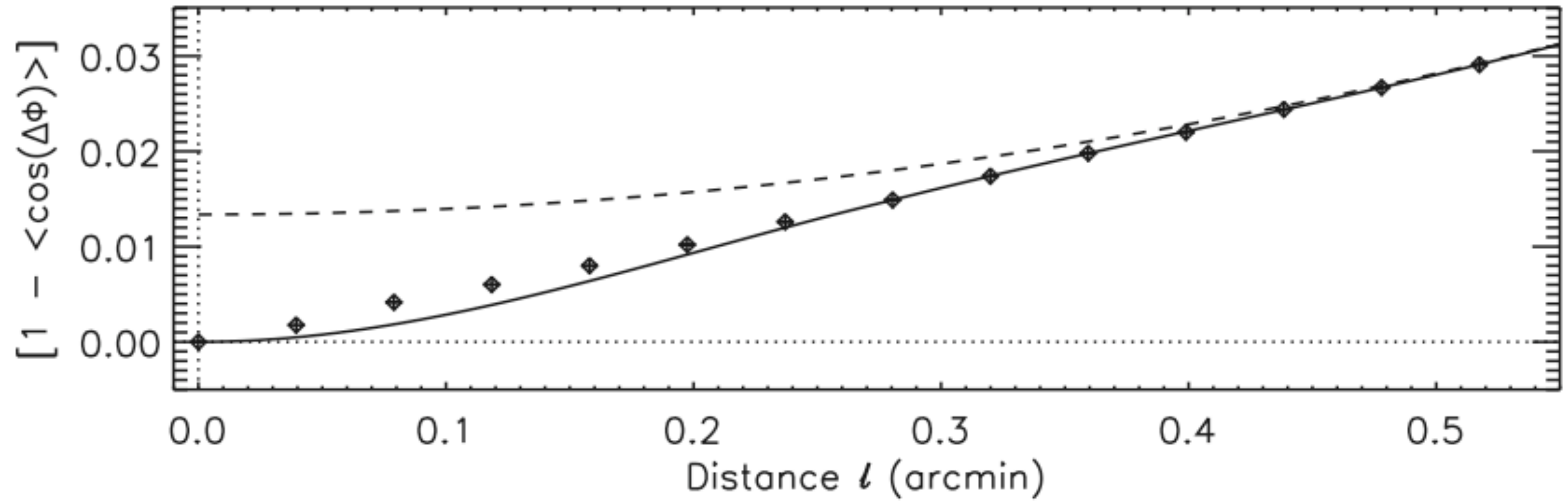


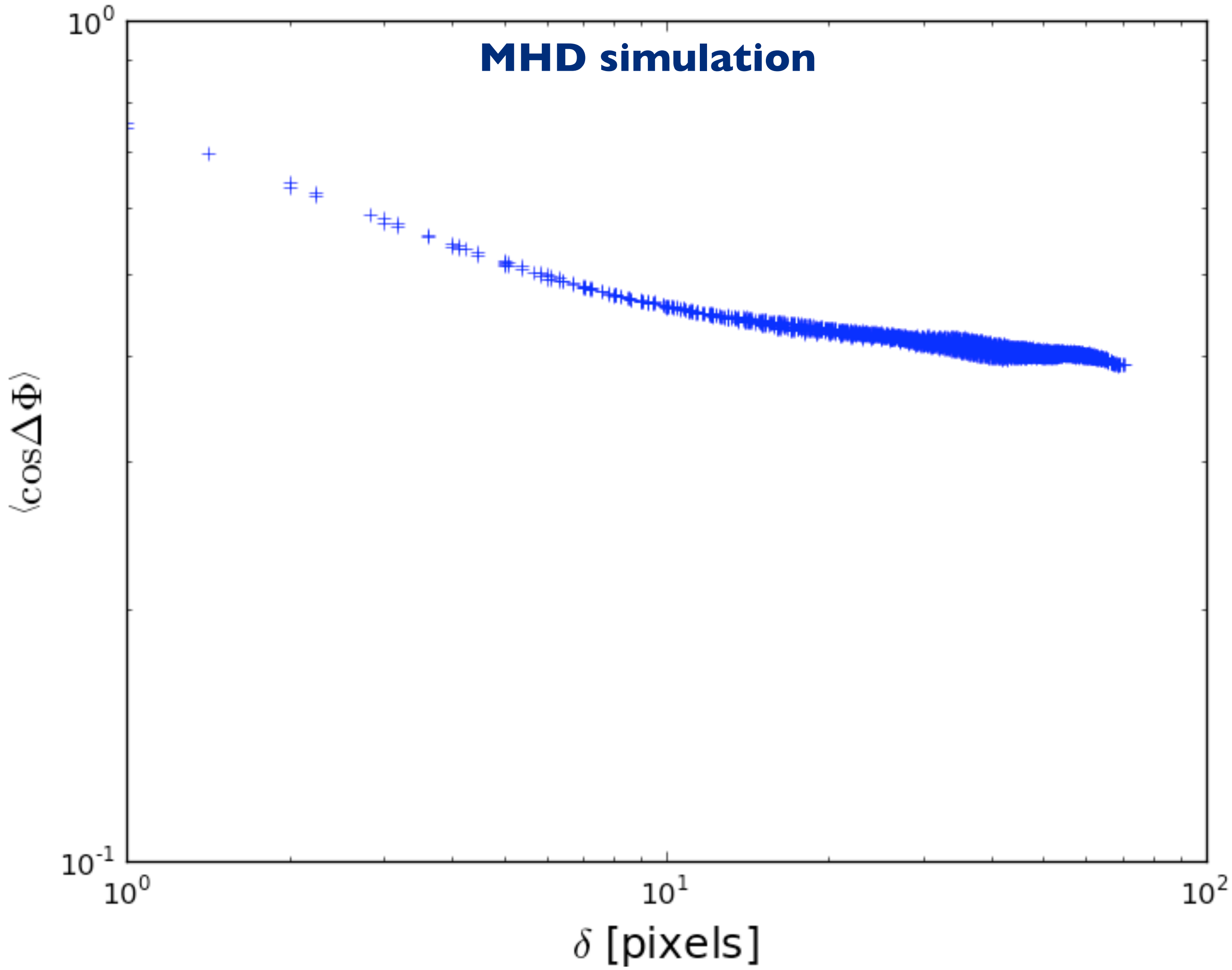
Hildebrand-Houde approach

(Hildebrand et al. 2009, Houde et al. 2009, 2011)

$$\langle\cos[\Delta\Phi(\boldsymbol{\ell})]\rangle = \frac{\langle \mathbf{B}(\mathbf{x})\cdot\mathbf{B}(\mathbf{x}+\boldsymbol{\ell})\rangle}{[\langle B^2(\mathbf{x})\rangle\langle B^2(\mathbf{x}+\boldsymbol{\ell})\rangle]^{1/2}}$$

350μm SHARP data for OMC-1





Questions...

- Noise
- Heterogeneous dust properties and alignment mechanisms
- Density-magnetic field correlations
- Effect of cloud depth or several clouds on the LOS
- Effect of strong density fluctuations
- Houde-Hildebrand approach requires scale separation